HEADS-UP
REACH FOR THE STARS

IGCSE

ADDITIONAL MATHEMATICS

TOPICAL PRACTICE QUESTIONS

PAPER 2

VOL. 2

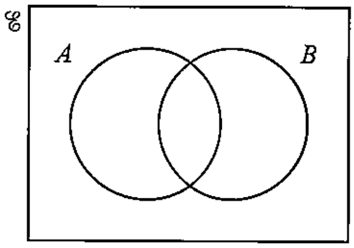
2012-2015

STUDENT COPY

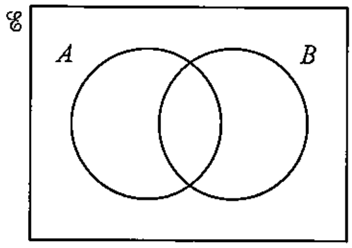
2020 EDITION

1 By shading the Venn diagrams below, investigate whether each of the following statements is true or false. State your conclusions clearly.

(i) $A \cap B' = (A' \cap B)'$ [2]

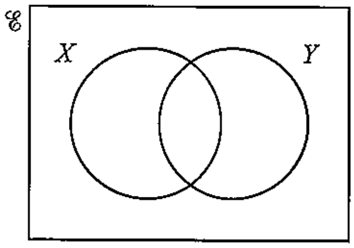


$A \cap B'$

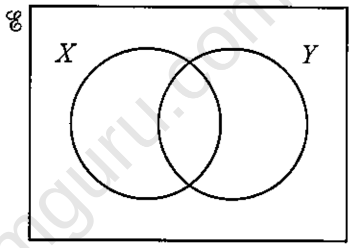


$(A' \cap B)'$

(ii) $X \cap Y = X' \cup Y'$ [2]

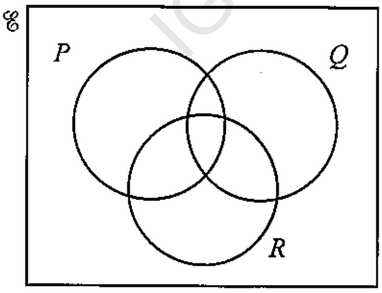


$X \cap Y$

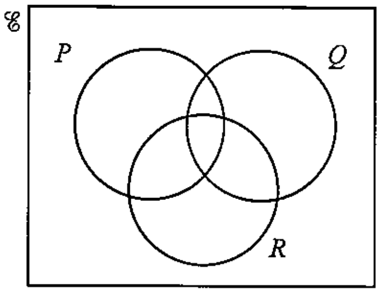


$X' \cup Y'$

(iii) $(P \cap Q) \cup (Q \cap R) = Q \cap (P \cup R)$ [2]



$(P \cap Q) \cup (Q \cap R)$



$Q \cap (P \cup R)$

Paper 2 - May Jun 2012 Code 21,23

2 It is given that P is the set of prime numbers, S is the set of square numbers and N is the set of numbers between 10 and 90. Write each of the following statements using set notation.

(i) 7 is a prime number. [1]

(ii) 8 is not a square number. [1]

(iii) There are 6 square numbers between 10 and 90. [1]

Paper 2 - May June 2012 Code 22

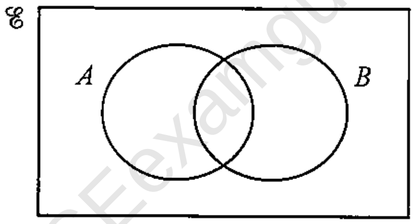
3 (a) It is given that $\mathscr{E}$ is the set of integers, P is the set of prime numbers between 10 and 50, F is the set of multiples of 5, and T is the set of multiples of 10. Write the following statements using set notation.

(i) There are 11 prime numbers between 10 and 50. [1]

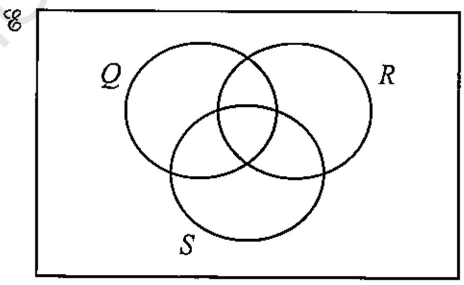
(ii) 18 is not a multiple of 5. [1]

(iii) All multiples of 10 are multiples of 5. [1]

(b) (i) In the Venn diagram below shade the region that represents $(A' \cap B) \cup (A \cap B')$. [1]



(ii) In the Venn diagram below shade the region that represents $Q \cap (R \cup S')$. [1]



4 It is given that $x \in \mathbb{R}$ and that $\mathscr{E} = \{x : -5 < x < 12\}$,
$S = \{x : 5x + 24 > x^2\}$,
$T = \{x : 2x + 7 > 15\}$.

Find the values of x such that

(i) $x \in S$, [3]

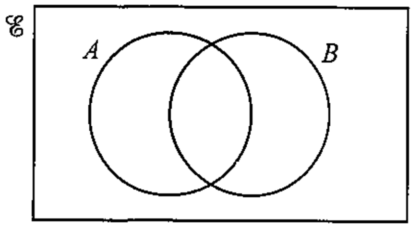
(ii) $x \in S \cup T$, [2]

(iii) $x \in (S \cap T)'$. [3]

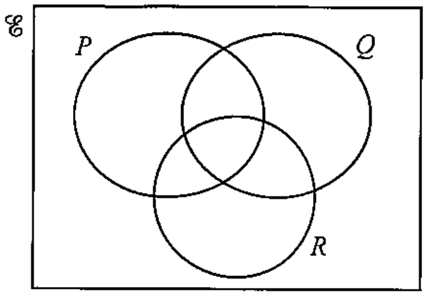
Paper 2 - May June 2013 Code 21,23

Paper 2 - Oct Nov 2013 Code 23

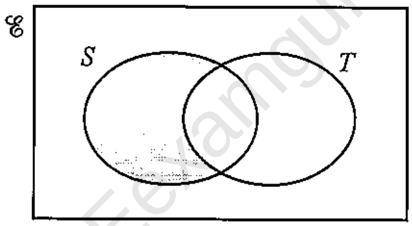
5 **(a)** **(i)** In the Venn diagram below shade the region that represents $(A \cup B)'$. [1]



(ii) In the Venn diagram below shade the region that represents $P \cap Q \cap R'$. [1]



(b) Express, in set notation, the set represented by the shaded region. [1]



Answer ....

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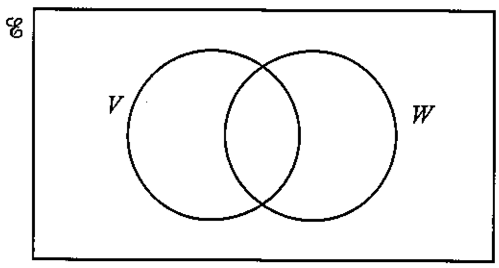
(c) The universal set $\mathscr{E}$ and the sets V and W are such that $n(\mathscr{E}) = 40$, $n(V) = 18$ and $n(W) = 14$. Given that $n(V \cap W) = x$ and $n((V \cup W)') = 3x$ find the value of x.

You may use the Venn diagram below to help you. [3]



6 **(a)** Illustrate the following statements using the Venn diagrams below.

(i) $A \cup B = A$ **(ii)** $A \cap B \cap C = \varnothing$ [2]

$\mathscr{E}$ $\mathscr{E}$

(b) It is given that $\mathscr{E}$ is the set of integers between 1 and 100 inclusive. S and C are subsets of $\mathscr{E}$, where S is the set of square numbers and C is the set of cube numbers. Write the following statements using set notation.

(i) 50 is not a cube number. [1]

(ii) 64 is both a square number and a cube number. [1]

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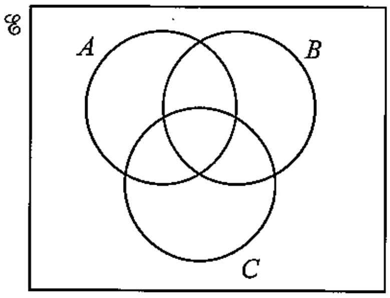
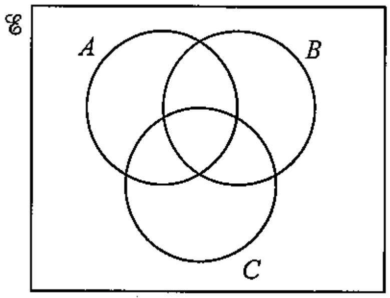
(iii) There are 90 integers between 1 and 100 inclusive which are not square numbers. [1]

7 **(a)** On each of the Venn diagrams below shade the region which represents the given set.



$(A \cap B) \cup C'$



$A' \cap (B \cup C)$ [2]

(b) In a year group of 98 pupils, F is the set of pupils who play football and H is the set of pupils who play hockey. There are 60 pupils who play football and 50 pupils who play hockey. The number that play both sports is x and the number that play neither is $30 - 2x$. Find the value of x. [3]

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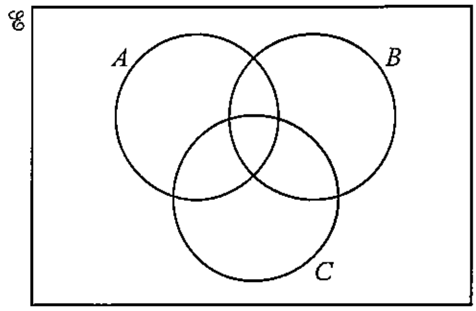
8



The Venn diagram above shows the sets A, B and C. It is given that

$n(A \cup B \cup C) = 48$,
$n(A) = 30$, $n(B) = 25$, $n(C) = 15$,
$n(A \cap B) = 7$, $n(B \cap C) = 6$, $n(A' \cap B \cap C') = 16$.

(i) Find the value of x, where $x = n(A \cap B \cap C)$. [3]

(ii) Find the value of y, where $y = n(A \cap B' \cap C)$. [3]

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(iii) Hence show that $A' \cap B' \cap C = \varnothing$. [1]

8 The line $y = 2x + 10$ intersects the curve $2x^2 + 3xy - 5y + y^2 = 218$ at the points A and B. Find the equation of the perpendicular bisector of AB. [9]

Oct Nov 2011 Code 23

9 Find the values of m for which the line $y = mx - 5$ is a tangent to the curve $y = x^2 + 3x + 4$. [5]

May Jun 2012 Code 22

10 Find the set of values of m for which the line $y = mx + 2$ does not meet the curve $y = mx^2 + 7x + 11$. [6]

May Jun 2014 Code 22

11 The line $y = 2x - 8$ cuts the curve $2x^2 + y^2 - 5xy + 32 = 0$ at the points A and B. Find the length of the line AB. [7]

May Jun 2013 Code 21,23

12 Find the set of values of k for which the line $y = 3x - k$ does not meet the curve $y = kx^2 + 11x - 6$. [6]

Oct Nov 2013 Code 23

13 The line $4x + y = 16$ intersects the curve $\frac{4}{x} - \frac{8}{y} = 1$ at the points A and B. The x-coordinate of A is less than the x-coordinate of B. Given that the point C lies on the line AB such that $AC : CB = 1 : 2$, find the coordinates of C. [8]

Oct Nov 2013 Code 23

14 Find the values of k for which the line $y + kx - 2 = 0$ is a tangent to the curve $y = 2x^2 - 9x + 4$. [5]

May Jun 2014 Code 22

15 The line $y = x - 5$ meets the curve $x^2 + y^2 + 2x - 35 = 0$ at the points A and B. Find the exact length of AB. [6]

6 Find the coordinates of the points of intersection of the curve $\frac{8}{x}-\frac{10}{y}=1$ and the line $x+y=9$. [6]

May Jun 2014 Code 23

17 **(i)** Calculate the coordinates of the points where the line $y = x + 2$ cuts the curve $x^2 + y^2 = 10$. [4]

(ii) Find the exact values of m for which the line $y = mx + 5$ is a tangent to the curve $x^2 + y^2 = 10$. [4]

Oct Nov 2014 Code 21,22

18 Solve the simultaneous equations

$$2x^2 + 3y^2 = 7xy,$$
$$x + y = 4.$$

[5]

May Jun 2015 Code 22

19 Find the values of k for which the line $y = 2x + k + 2$ cuts the curve $y = 2x^2 + (k+2)x + 8$ in two distinct points. [6]

Paper 2 - Oct Nov 2015 Code 23

20 Solve the following simultaneous equations, giving your answers for both x and y in the form $a + b\sqrt{3}$, where a and b are integers.

$$2x + y = 9$$

$$\sqrt{3}x + 2y = 5$$

[5]

Paper 2 - Oct Nov 2015 Code 23

21 Find the coordinates of the points of intersection of the curve $4 + \frac{5}{y} + \frac{3}{x} = 0$ and the line $y = 15x + 10$. [6]

0606/22/M/J/16

1 A cuboid has a square base of side $(2 + \sqrt{3})$ cm and a volume of $(16 + 9\sqrt{3})$ cm^3. Without using a calculator, find the height of the cuboid in the form $(a + b\sqrt{3})$ cm, where a and b are integers. [4]

May Jun 2012 Code 21,23

2 (a) Solve the equation $3^{2x} = 1000$, giving your answer to 2 decimal places. [2]

(b) Solve the equation $\frac{36^{2y-5}}{6^{3y}} = \frac{6^{2y-1}}{216^{y+6}}$. [4]

May Jun 2012 Code 21,23

3 (i) Given that $\frac{2^{x-3}}{8^{2y-3}} = 16^{x-y}$, show that $3x + 2y = 6$. [2]

(ii) Given also that $\frac{5^y}{125^{x-2}} = 25$, find the value of x and of y. [4]

Oct Nov 2012 Code 22

4 Without using a calculator, simplify $\frac{(3\sqrt{3}-1)^2}{2\sqrt{3}-3}$, giving your answer in the form $\frac{a\sqrt{3}+b}{3}$, where a and b are integers. [4]

Oct Nov 2012 Code 23

5 **(a)** Solve $(2^{x-2})^{\frac{1}{2}} = 100$, giving your answer to 1 decimal place. [3]

(b) Solve $\log_y 2 = 3 - \log_y 256$. [3]

(c) Solve $\frac{6^{5z-2}}{36^z} = \frac{216^{z-1}}{36^{3-z}}$. [4]

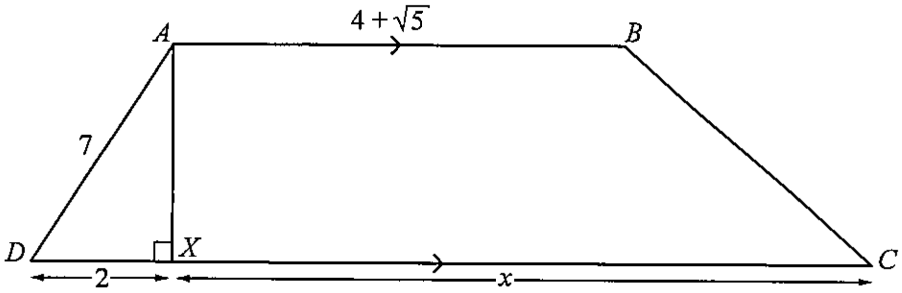
Oct Nov 2012 Code 23

6 (a) Solve the equation $3^{p+1} = 0.7$, giving your answer to 2 decimal places. [3]

(b) Express $\dfrac{y \times (4x^3)^2}{\sqrt{8y^3}}$ in the form $2^a \times x^b \times y^c$, where a, b and c are constants. [3]

May Jun 2013 Code 22

7 **Calculators must not be used in this question.**



The diagram shows a trapezium $ABCD$ in which $AD = 7\text{cm}$ and $AB = (4+\sqrt{5})\text{cm}$. AX is perpendicular to DC with $DX = 2\text{cm}$ and $XC = x\text{cm}$. Given that the area of trapezium $ABCD$ is $15(\sqrt{5}+2)\text{cm}^2$, obtain an expression for x in the form $a + b\sqrt{5}$, where a and b are integers. [6]

May Jun 2013 Code 22

8 **Do not use a calculator in this question.**

Express $\dfrac{(4\sqrt{5}-2)^2}{\sqrt{5}-1}$ in the form $p\sqrt{5}+q$, where p and q are integers. [4]

Oct Nov 2013 Code 21,22

9 Given that $\log_p X = 5$ and $\log_p Y = 2$, find

(i) $\log_p X^2$, [1]

(ii) $\log_p \frac{1}{X}$, [1]

(iii) $\log_{XY} p$. [2]

Oct Nov 2013 Code 21,22

10 Solve the simultaneous equations

$$\frac{4^x}{256^y} = 1024,$$
$$3^{2x} \times 9^y = 243.$$ [5]

Oct Nov 2013 Code 21,22

11 **Without using a calculator**, express $6(1+\sqrt{3})^{-2}$ in the form $a + b\sqrt{3}$, where a and b are integers to be found. [4]

May Jun 2014 Code 21

12 **(a)** Solve $2^{x^2-5x} = \frac{1}{64}$. [4]

(b) By changing the base of $\log_{2a}4$, express $(\log_{2a}4)(1+\log_a 2)$ as a single logarithm to base a. [4]

May Jun 2014 Code 21

13 **Without using a calculator**, express $\frac{(2+\sqrt{5})^2}{\sqrt{5}-1}$ in the form $a + b\sqrt{5}$, where a and b are constants to be found. [4]

May Jun 2014 Code 22

14 **(a)** **(i)** State the value of u for which $\lg u = 0$. [1]

(ii) Hence solve $\lg|2x + 3| = 0$. [2]

(b) Express $2\log_3 15 - (\log_a 5)(\log_3 a)$, where $a > 1$, as a single logarithm to base 3. [4]

May Jun 2014 Code 22

15 Using the substitution $u = \log_3 x$, solve, for x, the equation $\log_3 x - 12\log_x 3 = 4$. [5]

May Jun 2014 Code 23

16 Do not use a calculator in this question.

(i) Show that $(2\sqrt{2}+4)^2 - 8(2\sqrt{2}+3) = 0$. [2]

(ii) Solve the equation $(2\sqrt{2}+3)x^2 - (2\sqrt{2}+4)x + 2 = 0$, giving your answer in the form $a + b\sqrt{2}$ where a and b are integers. [3]

May Jun 2014 Code 23

17 Solve the following simultaneous equations.

$$\log_2(x+3) = 2 + \log_2 y$$

$$\log_2(x+y) = 3$$ [5]

Oct Nov 2014 Code 21,22

18 Integers a and b are such that $(a+3\sqrt{5})^2 + a - b\sqrt{5} = 51$. Find the possible values of a and the corresponding values of b. [6]

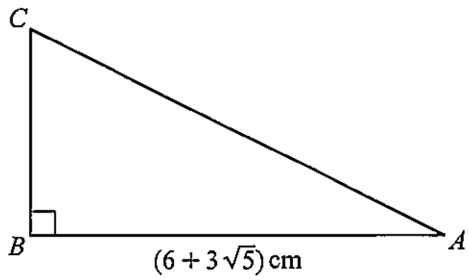
Oct Nov 2014 Code 21,22

19 **(a)** Write $\log_{27}x$ as a logarithm to base 3. [2]

(b) Given that $\log_a y = 3(\log_a 15 - \log_a 3) + 1$, express y in terms of a. [3]

May Jun 2015 Code 21,23

20 **Do not use a calculator in this question.**



The diagram shows the right-angled triangle ABC, where $AB = (6 + 3\sqrt{5})$cm and angle $B = 90°$. The area of this triangle is $\left(\frac{36+15\sqrt{5}}{2}\right)\text{cm}^2$.

(i) Find the length of the side BC in the form $(a + b\sqrt{5})$cm, where a and b are integers. [3]

(ii) Find $(AC)^2$ in the form $(c + d\sqrt{5})\text{cm}^2$, where c and d are integers. [2]

May Jun 2015 Code 22

21 **(a)** Solve $6^{x-2} = \frac{1}{4}$. [2]

(b) Solve $\log_a 2y^2 + \log_a 8 + \log_a 16y - \log_a 64y = 2\log_a 4$. [4]

May Jun 2015 Code 22

22 **(a)** Solve the following equations to find p and q.

$$8^{q-1} \times 2^{2p+1} = 4^7$$
$$9^{p-4} \times 3^q = 81$$ [4]

(b) Solve the equation $\lg(3x-2)+\lg(x+1)=2-\lg 2$. [5]

Paper 2 - Oct Nov 2015 Code 21,22

23 Solve the following equation.

$$\log_2(29x - 15) = 3 + \frac{2}{\log_x 2}$$ [5]

Paper 2 - Oct Nov 2015 Code 23

1 It is given that $x - 2$ is a factor of $f(x) = x^3 + kx^2 - 8x - 8$.

(i) Find the value of the integer k. [2]

(ii) Using your value of k, find the non-integer roots of the equation $f(x) = 0$ in the form $a \pm \sqrt{b}$, where a and b are integers. [5]

May Jun 2012 Code 22

2 (i) The remainder when the expression $x^3 + 9x^2 + bx + c$ is divided by $x - 2$ is twice the remainder when the expression is divided by $x - 1$. Show that $c = 24$. [5]

(ii) Given that $x + 8$ is a factor of $x^3 + 9x^2 + bx + 24$, show that the equation $x^3 + 9x^2 + bx + 24 = 0$ has only one real root. [4]

Oct Nov 2012 Code 21

3 The function $f(x) = x^3 + x^2 + ax + b$ is divisible by $x - 3$ and leaves a remainder of 20 when divided by $x + 1$.

(i) Show that $b = 6$ and find the value of a. [4]

(ii) Using your value of a and taking b as 6, find the non-integer roots of the equation $f(x) = 0$ in the form $p \pm \sqrt{q}$, where p and q are integers. [5]

May Jun 2013 Code 21,23

4 The expression $2x^3 + ax^2 + bx + 21$ has a factor $x + 3$ and leaves a remainder of 65 when divided by $x - 2$.

(i) Find the value of a and of b. [5]

(ii) Hence find the value of the remainder when the expression is divided by $2x + 1$. [2]

Oct Nov 2013 Code 23

5 The expression $2x^3 + ax^2 + bx + 12$ has a factor $x - 4$ and leaves a remainder of -12 when divided by $x - 1$. Find the value of each of the constants a and b. [5]

May Jun 2014 Code 21

6 **(i)** Given that $x + 1$ is a factor of $3x^3 - 14x^2 - 7x + d$, show that $d = 10$. [1]

(ii) Show that $3x^3 - 14x^2 - 7x + 10$ can be written in the form $(x + 1)(ax^2 + bx + c)$, where a, b and c are constants to be found. [2]

(iii) Hence solve the equation $3x^3 - 14x^2 - 7x + 10 = 0$. [2]

May Jun 2014 Code 22

7 **(i)** Show that $x-2$ is a factor of $3x^3-14x^2+32$. [1]

(ii) Hence factorise $3x^3-14x^2+32$ completely. [4]

Oct Nov 2014 Code 21,22

8 The expression $f(x) = 3x^3 + 8x^2 - 33x + p$ has a factor of $x - 2$.

(i) Show that $p = 10$ and express $f(x)$ as a product of a linear factor and a quadratic factor. [4]

(ii) Hence solve the equation $f(x) = 0$. [2]

Oct Nov 2014 Code 23

9 **(i)** Show that $x = -2$ is a root of the polynomial equation $15x^3 + 26x^2 - 11x - 6 = 0$. [1]

(ii) Find the remainder when $15x^3 + 26x^2 - 11x - 6$ is divided by $x - 3$. [2]

(iii) Find the value of p and of q such that $15x^3 + 26x^2 - 11x - 6$ is a factor of $15x^4 + px^3 - 37x^2 + qx + 6$. [4]

May Jun 2015 Code 22

10 It is given that $f(x) = 4x^3 - 4x^2 - 15x + 18$.

(i) Show that $x + 2$ is a factor of $f(x)$. [1]

(ii) Hence factorise $f(x)$ completely and solve the equation $f(x) = 0$. [4]

Paper 2 - Oct Nov 2015 Code 21,22

11 The roots of the equation $x^3 + ax^2 + bx + c = 0$ are 1, 3 and 3. Show that $c = -9$ and find the value of a and of b. [4]

Paper 2 - Oct Nov 2015 Code 23

1 **(i)** Given that $\mathbf{A} = \begin{pmatrix} 4 & -3 \\ 2 & 5 \end{pmatrix}$, find the inverse matrix $\mathbf{A}^{-1}$. [2]

(ii) Use your answer to part **(i)** to solve the simultaneous equations

$$4x - 3y = -10,$$
$$2x + 5y = 21.$$

[2]

May Jun 2012 Code 21,23

2 In a competition the contestants search for hidden targets which are classed as difficult, medium or easy. In the first round, finding a difficult target scores 5 points, a medium target 3 points and an easy target 1 point. The number of targets found by the two contestants, Claire and Denise, are shown in the table.

Contestant \ Target	Difficult	Medium	Easy
Claire	4	1	7
Denise	2	5	1

In the second round, finding a difficult target scores 8 points, a medium target 4 points and an easy target 2 points. In the second round Claire finds 2 difficult, 5 medium and 2 easy targets whilst Denise finds 4 difficult, 3 medium and 6 easy targets.

(i) Write down the sum of two matrix products which, on evaluation, would give the total score for each contestant. [3]

(ii) Use matrix multiplication and addition to calculate the total score for each contestant. [2]

May Jun 2012 Code 22

3 It is given that $\mathbf{A} = \begin{pmatrix} 4 & -2 \\ 8 & -3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 0 & 4 \\ 5 & -1 & 4 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$.

(i) Calculate **ABC**. [4]

(ii) Calculate $\mathbf{A}^{-1}\mathbf{B}$. [4]

Oct Nov 2012 Code 21

4 (i) Given that $\mathbf{A} = \begin{pmatrix} 7 & 8 \\ 4 & 6 \end{pmatrix}$, find the inverse matrix, $\mathbf{A}^{-1}$. [2]

(ii) Use your answer to part (i) to solve the simultaneous equations

$$7x + 8y = 39,$$
$$4x + 6y = 23.$$ [2]

Oct Nov 2012 Code 23

5 In a motor racing competition, the winning driver in each race scores 5 points, the second and third placed drivers score 3 and 1 points respectively. Each team has two members. The results of the drivers in one team, over a number of races, are shown in the table below.

Driver	1^{st} place	2^{nd} place	3^{rd} place
Alan	3	1	4
Brian	1	4	0

(i) Write down two matrices whose product under matrix multiplication will give the number of points scored by each of the drivers. Hence calculate the number of points scored by Alan and by Brian. [3]

(ii) The points scored by Alan and by Brian are added to give the number of points scored by the team. Using your answer to part **(i)**, write down two matrices whose product would give the number of points scored by the team. [1]

May Jun 2014 Code 23

6 (a) Find the matrix **A** if $4\mathbf{A} + 5\begin{pmatrix} 4 & 0 & -1 \\ 3 & -2 & 5 \end{pmatrix} = \begin{pmatrix} 52 & -8 & 19 \\ 31 & 2 & 65 \end{pmatrix}$. [2]

(b) $\mathbf{P} = \begin{pmatrix} 30 & 25 & 65 \\ 70 & 15 & 80 \\ 50 & 40 & 30 \\ 40 & 20 & 75 \end{pmatrix}$ $\quad \mathbf{Q} = \begin{pmatrix} 650 & 500 & 450 & 225 \end{pmatrix}$

The matrix **P** represents the number of 4 different televisions that are on sale in each of 3 shops. The matrix **Q** represents the value of each television in dollars.

(i) State, without evaluation, what is represented by the matrix **QP**. [1]

(ii) Given that the matrix $\mathbf{R} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, state, without evaluation, what is represented by the matrix **QPR**. [1]

May Jun 2015 Code 21,23

7 The table shows the number of passengers in Economy class and in Business class on 3 flights from London to Paris. The table also shows the departure times for the 3 flights and the cost of a single ticket in each class.

Departure time	Number of passengers in Economy class	Number of passengers in Business class
0930	60	50
1330	70	52
1545	58	34
Single ticket price (£)	120	300

(i) Write down a matrix, **P**, for the numbers of passengers and a matrix, **Q**, of single ticket prices, such that the matrix product **QP** can be found. [2]

(ii) Find the matrix product **QP**. [2]

(iii) Given that $\mathbf{R} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, explain what information is found by evaluating the matrix product **QPR**. [1]

May Jun 2015 Code 22

8 (a) Given that $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 5 \\ 7 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & -2 & 4 \\ -2 & 3 & 0 \end{pmatrix}$, calculate $\mathbf{2BA}$. [3]

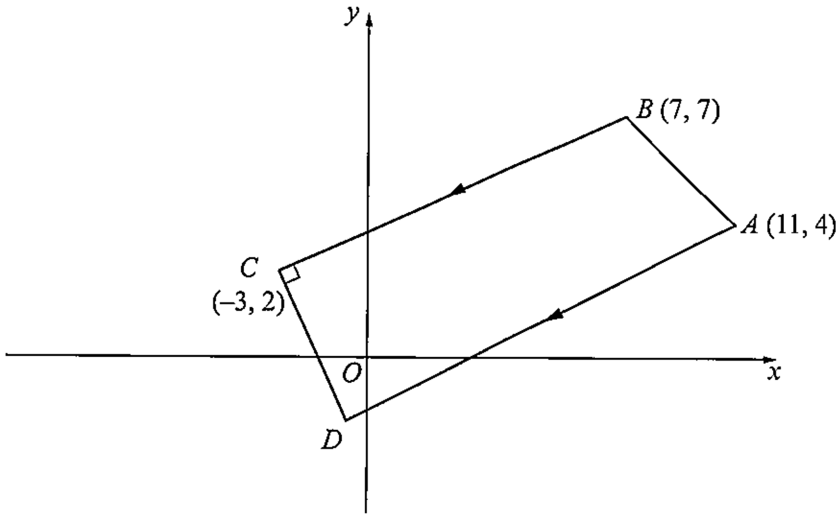
(b) The matrices **C** and **D** are given by $\mathbf{C} = \begin{pmatrix} 1 & 2 \\ -1 & 6 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix}$.

(i) Find $\mathbf{C}^{-1}$. [2]

(ii) Hence find the matrix **X** such that $\mathbf{CX} + \mathbf{D} = \mathbf{I}$, where **I** is the identity matrix. [3]

Paper 2 - Oct Nov 2015 Code 21,22

1 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a trapezium $ABCD$ with vertices $A(11, 4)$, $B(7, 7)$, $C(-3, 2)$ and D. The side AD is parallel to BC and the side CD is perpendicular to BC. Find the area of the trapezium $ABCD$. [9]

May Jun 2012 Code 21,23

2 **Solutions to this question by accurate drawing will not be accepted.**

The points A (4, 5), B(–2, 3), C(1, 9) and D are the vertices of a trapezium in which BC is parallel to AD and angle BCD is 90°. Find the area of the trapezium. [8]

Oct Nov 2012 Code 23

3 **Solutions to this question by accurate drawing will not be accepted.**

The points $A(-6, 2)$, $B(2, 6)$ and C are the vertices of a triangle.

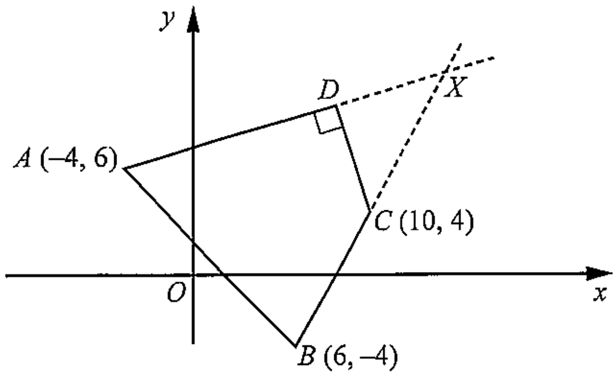
(i) Find the equation of the line AB in the form $y = mx + c$. [2]

(ii) Given that angle $ABC = 90°$, find the equation of BC. [2]

(iii) Given that the length of AC is 10 units, find the coordinates of each of the two possible positions of point C. [4]

May Jun 2013 Code 22

4 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a quadrilateral $ABCD$, with vertices $A(-4, 6)$, $B(6, -4)$, $C(10, 4)$ and D. The angle $ADC = 90°$. The lines BC and AD are extended to intersect at the point X.

(i) Given that C is the midpoint of BX, find the coordinates of D. [7]

(ii) Hence calculate the area of the quadrilateral $ABCD$. [2]

Oct Nov 2013 Code 23

5 **Solutions to this question by accurate drawing will not be accepted.**

The points $A(p, 1)$, $B(1, 6)$, $C(4, q)$ and $D(5, 4)$, where p and q are constants, are the vertices of a kite $ABCD$. The diagonals of the kite, AC and BD, intersect at the point E. The line AC is the perpendicular bisector of BD. Find

(i) the coordinates of E, [2]

(ii) the equation of the diagonal AC, [3]

(iii) the area of the kite $ABCD$. [3]

May Jun 2014 Code 21

6 **Solutions to this question by accurate drawing will not be accepted.**
The points $A(2, 11)$, $B(-2, 3)$ and $C(2, -1)$ are the vertices of a triangle.

(i) Find the equation of the perpendicular bisector of AB. [4]

The line through A parallel to BC intersects the perpendicular bisector of AB at the point D.

(ii) Find the area of the quadrilateral $ABCD$. [6]

May Jun 2014 Code 23

7 Points A and B have coordinates $(-2, 10)$ and $(4, 2)$ respectively. C is the mid-point of the line AB. Point D is such that $\overrightarrow{CD} = \begin{pmatrix} 12 \\ 9 \end{pmatrix}$.

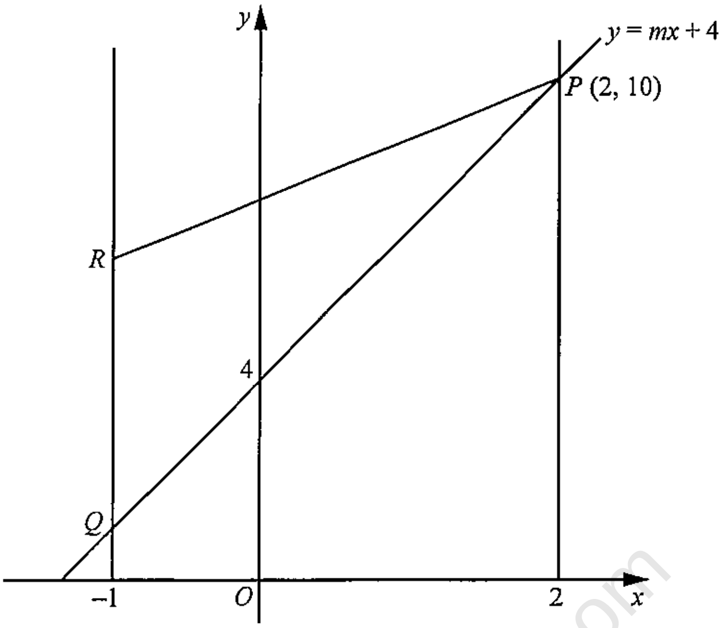
(i) Find the coordinates of C and of D. [3]

(ii) Show that CD is perpendicular to AB. [3]

(iii) Find the area of triangle ABD. [2]

Oct Nov 2014 Code 23

8 **Solutions by accurate drawing will not be accepted.**



The line $y = mx + 4$ meets the lines $x = 2$ and $x = -1$ at the points P and Q respectively. The point R is such that QR is parallel to the y-axis and the gradient of RP is 1. The point P has coordinates (2, 10).

(i) Find the value of m. [2]

(ii) Find the y-coordinate of Q. [1]

(iii) Find the coordinates of R. [2]

(iv) Find the equation of the line through P, perpendicular to PQ, giving your answer in the form $ax + by = c$, where a, b and c are integers. [3]

(v) Find the coordinates of the midpoint, M, of the line PQ. [2]

(vi) Find the area of triangle QRM. [2]

May Jun 2015 Code 22

9 **Solutions to this question by accurate drawing will not be accepted.**

Two points A and B have coordinates (–3, 2) and (9, 8) respectively.

(i) Find the coordinates of C, the point where the line AB cuts the y-axis. [3]

Paper 2 - Oct Nov 2015 Code 21,22

(ii) Find the coordinates of D, the mid-point of AB. [1]

(iii) Find the equation of the perpendicular bisector of AB. [2]

The perpendicular bisector of AB cuts the y-axis at the point E.

(iv) Find the coordinates of E. [1]

(v) Show that the area of triangle ABE is four times the area of triangle ECD. [3]

1 The table shows experimental values of variables x and y.

x	5	30	150	400
y	8.9	21.9	48.9	80.6

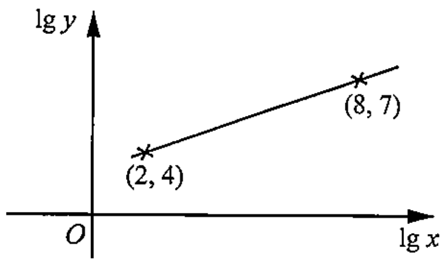
(i) By plotting a suitable straight line graph, show that y and x are related by the equation $y = ax^b$, where a and b are constants. [4]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) Estimate the value of y when $x = 100$. [2]

May Jun 2012 Code 22

2



The variables x and y are related in such a way that when lg y is plotted against lg x a straight line graph is obtained as shown in the diagram. The line passes through the points (2, 4) and (8, 7).

(i) Express y in terms of x, giving your answer in the form $y = ax^b$, where a and b are constants. [5]

Another method of drawing a straight line graph for the relationship $y = ax^b$, found in part **(i)**, involves plotting lg x on the horizontal axis and $\lg(y^2)$ on the vertical axis. For this straight line graph what is

(ii) the gradient, [1]

(iii) the intercept on the vertical axis? [1]

Oct Nov 2012 Code 22

3 The table shows experimental values of two variables x and y.

x	1	2	3	4
y	9.41	1.29	−0.69	−1.77

It is known that x and y are related by the equation $y = \frac{a}{x^2} + bx$, where a and b are constants.

(i) A straight line graph is to be drawn to represent this information. Given that x^2y is plotted on the vertical axis, state the variable to be plotted on the horizontal axis. [1]

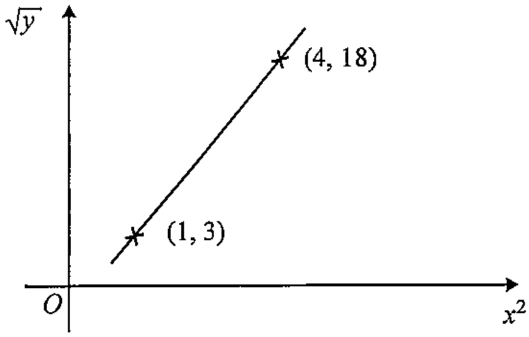
(ii) On the grid opposite, draw this straight line graph. [3]

(iii) Use your graph to estimate the value of a and of b. [3]

(iv) Estimate the value of y when x is 3.7. [2]

Oct Nov 2012 Code 23

4



Variables x and y are such that when $\sqrt{y}$ is plotted against x^2 a straight line graph passing through the points (1, 3) and (4, 18) is obtained. Express y in terms of x. [4]

May Jun 2013 Code 22

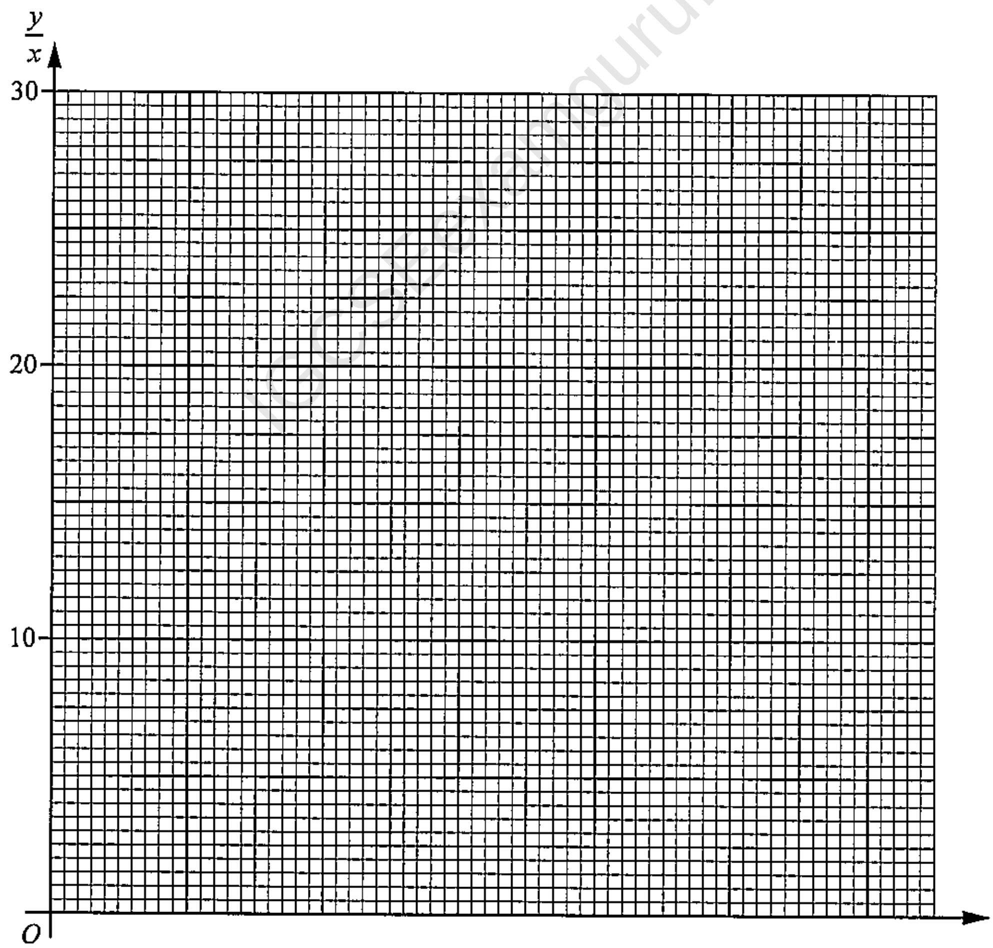
5 The table shows experimental values of two variables x and y.

x	2	4	6	8
y	9.6	38.4	105	232

It is known that x and y are related by the equation $y = ax^3 + bx$, where a and b are constants.

(i) A straight line graph is to be drawn for this information with $\frac{y}{x}$ on the vertical axis. State the variable which must be plotted on the horizontal axis. [1]

(ii) Draw this straight line graph on the grid below. [2]



(iii) Use your graph to estimate the value of a and of b. [3]

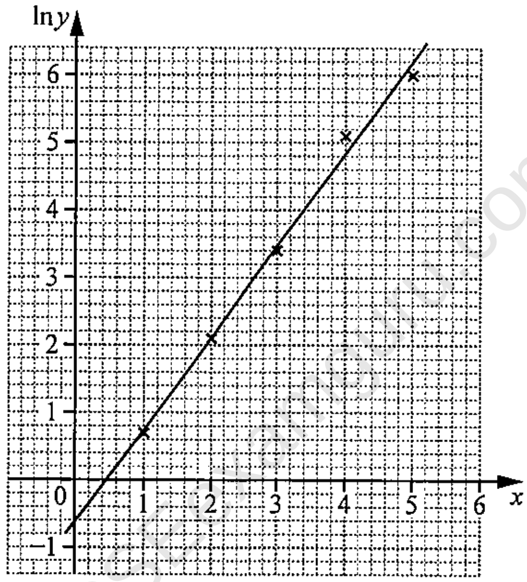
(iv) Estimate the value of x for which $2y = 25x$. [2]

Oct Nov 2013 Code 21,22

6 Two variables x and y are connected by the relationship $y = Ab^x$, where A and b are constants.

(i) Transform the relationship $y = Ab^x$ into a straight line form. [2]

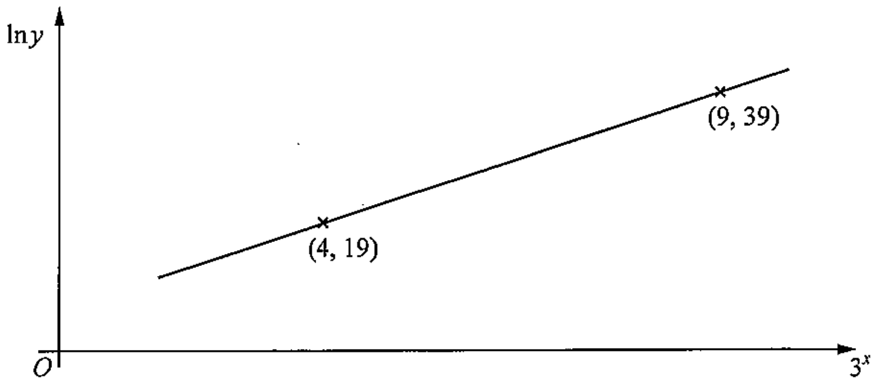
An experiment was carried out measuring values of y for certain values of x. The values of $\ln y$ and x were plotted and a line of best fit was drawn. The graph is shown on the grid below.



(ii) Use the graph to determine the value of A and the value of b, giving each to 1 significant figure. [4]

(iii) Find x when $y = 220$. [2]

7 Variables x and y are such that, when $\ln y$ is plotted against 3^x, a straight line graph passing through (4, 19) and (9, 39) is obtained.



(i) Find the equation of this line in the form $\ln y = m3^x + c$, where m and c are constants to be found. [3]

(ii) Find y when $x = 0.5$. [2]

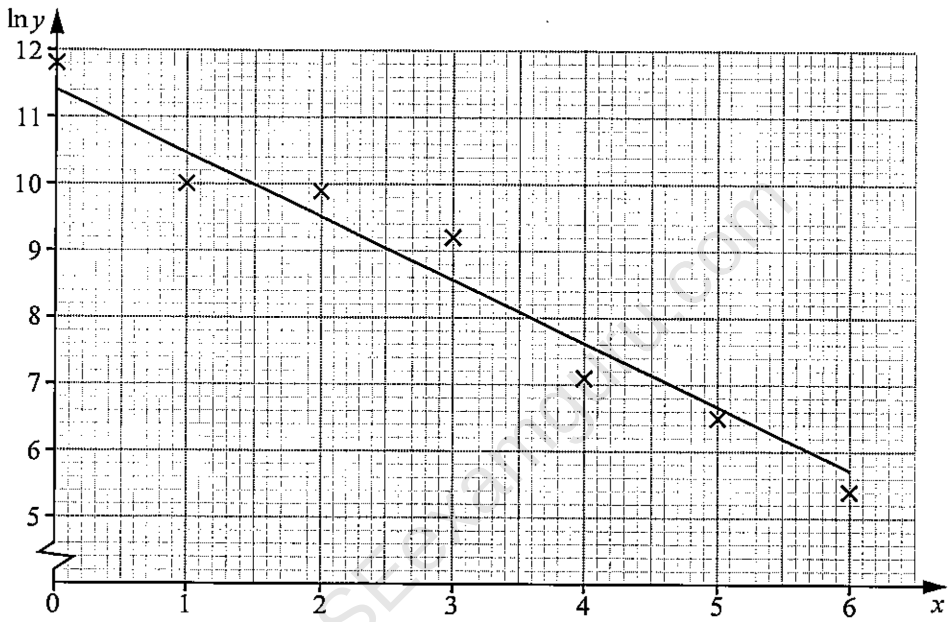
(iii) Find x when $y = 2000$. [3]

Oct Nov 2014 Code 23

8 The relationship between experimental values of two variables, x and y, is given by $y = Ab^x$, where A and b are constants.

(i) By transforming the relationship $y = Ab^x$, show that plotting $\ln y$ against x should produce a straight line graph. [2]

(ii) The diagram below shows the results of plotting $\ln y$ against x for 7 different pairs of values of variables, x and y. A line of best fit has been drawn.



By taking readings from the diagram, find the value of A and of b, giving each value correct to 1 significant figure. [4]

(iii) Estimate the value of y when $x = 2.5$. [2]

May Jun 2015 Code **21, 23**

9 The trees in a certain forest are dying because of an unknown virus.

The number of trees, N, surviving t years after the onset of the virus is shown in the table below.

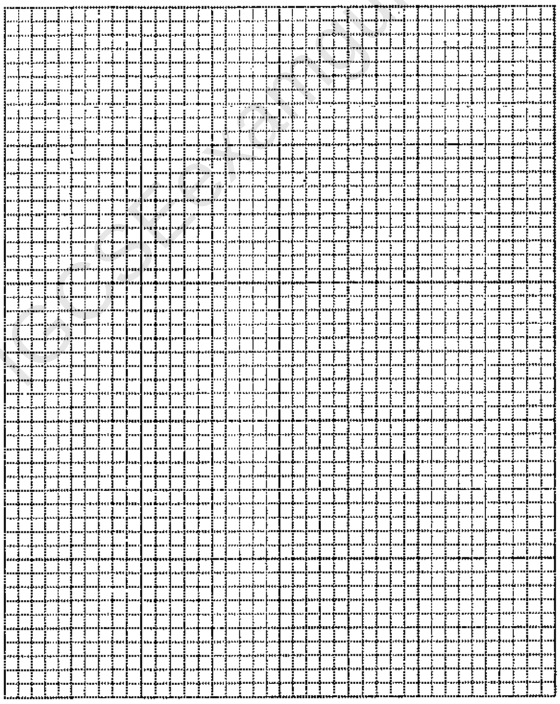
t	1	2	3	4	5	6
N	2000	1300	890	590	395	260

The relationship between N and t is thought to be of the form $N = Ab^{-t}$.

(i) Transform this relationship into straight line form. [1]

Paper 2 - Oct Nov 2015 Code 23

(ii) Using the given data, draw this straight line on the grid below. [3]



(iii) Use your graph to estimate the value of A and of b. [3]

If the trees continue to die in the same way, find

(iv) the number of trees surviving after 10 years, [1]

(v) the number of years taken until there are only 10 trees surviving. [2]

1

The equation of a curve is $y = 2x^2 - 20x + 37$.

(i) Express y in the form $a(x + b)^2 + c$, where a, b and c are integers. [3]

(ii) Write down the coordinates of the stationary point on the curve. [1]

A function f is defined by $f : x \mapsto 2x^2 - 20x + 37$ for $x > k$. Given that the function $f^{-1}(x)$ exists,

(iii) write down the least possible value of k, [1]

(iv) sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the axes provided, [2]

(v) obtain an expression for f^{-1}. [3]

May Jun 2012 Code 21,23

2 A function g is defined by $g : x \mapsto 5x^2 + px + 72$, where p is a constant. The function can also be written as $g : x \mapsto 5(x-4)^2 + q$.

(i) Find the value of p and of q. [3]

(ii) Find the range of the function g. [1]

(iii) Sketch the graph of the function on the axes provided. [2]

(iv) Given that the function h is defined by $h : x \mapsto \ln x$, where $x > 0$, solve the equation $gh(x) = 12$. [4]

May Jun 2012 Code 21,23

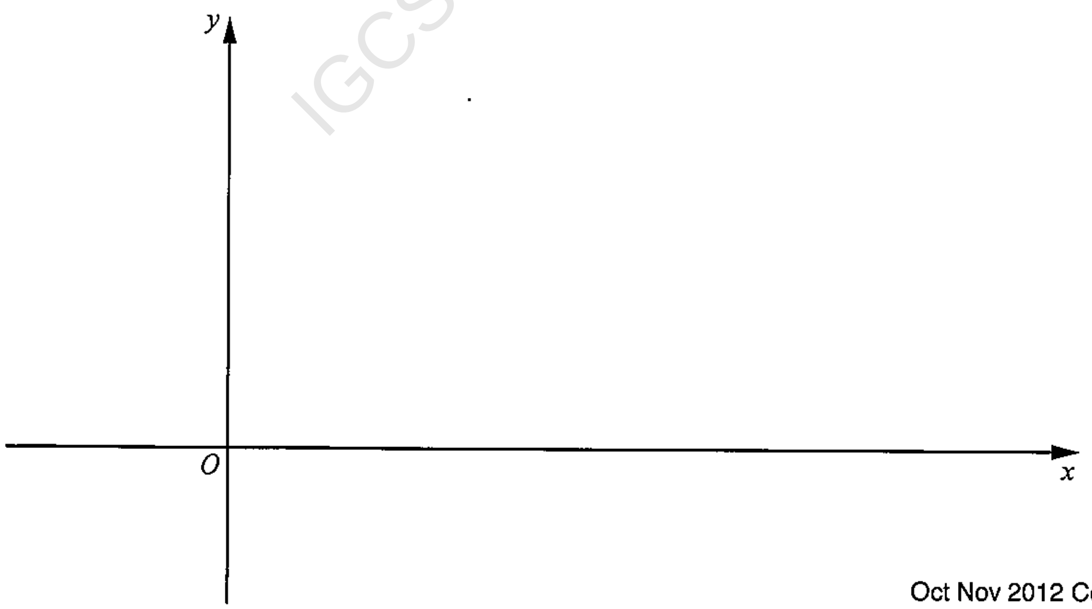
3 Solve the inequality $4x - 9 > 4x(5 - x)$. [4]

Oct Nov 2012 Code 21

4 It is given that $f(x) = 4 + 8x - x^2$.

(i) Find the value of a and of b for which $f(x) = a - (x + b)^2$ and hence write down the coordinates of the stationary point of the curve $y = f(x)$. [3]

(ii) On the axes below, sketch the graph of $y = f(x)$, showing the coordinates of the point where your graph intersects the y-axis. [2]



Oct Nov 2012 Code 21

5 Solve the equation $|7x + 5| = |3x - 13|$. [4]

Oct Nov 2012 Code 22

6 Solve the inequality $4x(4-x) > 7$. [4]

Oct Nov 2012 Code 22

7 Solve the equation $|5x + 7| = 13$. [3]

Oct Nov 2012 Code 23

8 (i) Express $4x^2 + 32x + 55$ in the form $(ax + b)^2 + c$, where a, b and c are constants and a is positive. [3]

The functions f and g are defined by

$f : x \mapsto 4x^2 + 32x + 55$ for $x > -4$,

$g : x \mapsto \dfrac{1}{x}$ for $x > 0$.

(ii) Find $f^{-1}(x)$. [3]

(iii) Solve the equation $fg(x) = 135$. [4]

Oct Nov 2012 Code 23

9 The functions h and k are defined by

$$h : x \mapsto \sqrt{2x - 7} \text{ for } x \geqslant c,$$

$$k : x \mapsto \frac{3x - 4}{x - 2} \text{ for } x > 2.$$

(i) State the least possible value of c. [1]

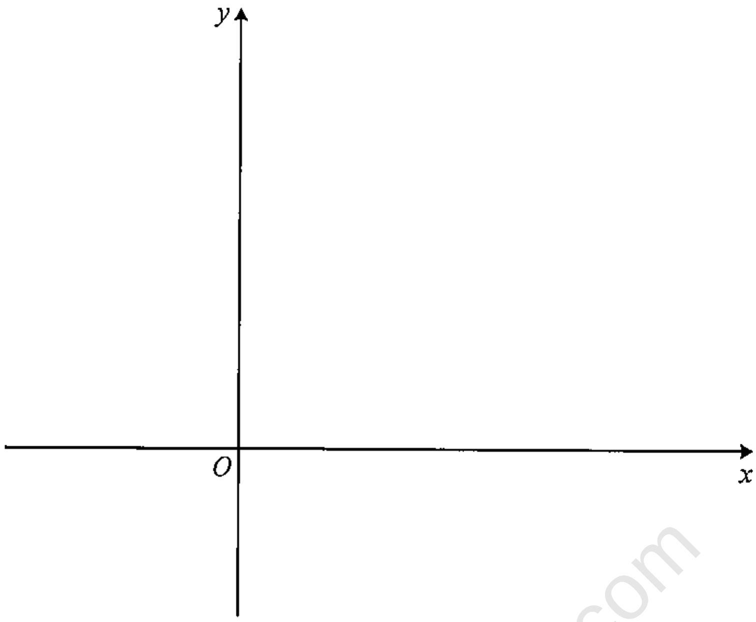
(ii) Find $h^{-1}(x)$. [2]

(iii) Solve the equation $k(x) = x$. [3]

(iv) Find an expression for the function k^2, in the form $k^2 : x \mapsto a + \frac{b}{x}$ where a and b are constants. [4]

Oct Nov 2012 Code 23

10 **(i)** Sketch the graph of $y = |4x - 2|$ on the axes below, showing the coordinates of the points where the graph meets the axes. [3]



(ii) Solve the equation $|4x - 2| = x$. [3]

May Jun 2013 Code 21,23

11 A one-one function f is defined by $f(x) = (x-1)^2 - 5$ for $x \geqslant k$.

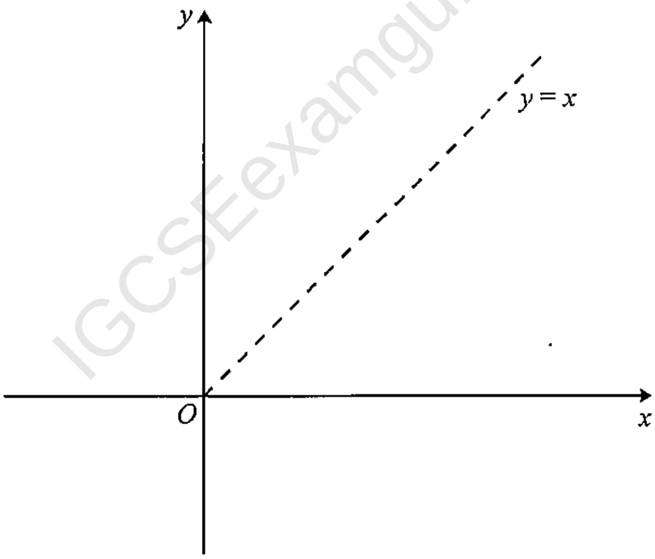
(i) State the least value that k can take. [1]

For this least value of k

(ii) write down the range of f, [1]

(iii) find $f^{-1}(x)$, [2]

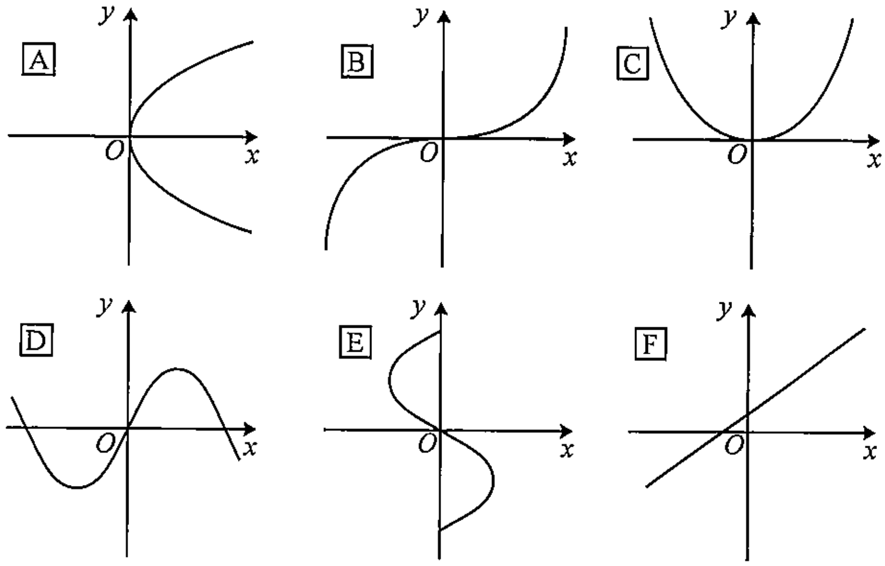
(iv) sketch and label, on the axes below, the graph of $y = f(x)$ and of $y = f^{-1}(x)$, [2]



(v) find the value of x for which $f(x) = f^{-1}(x)$. [2]

May Jun 2013 Code 21,23

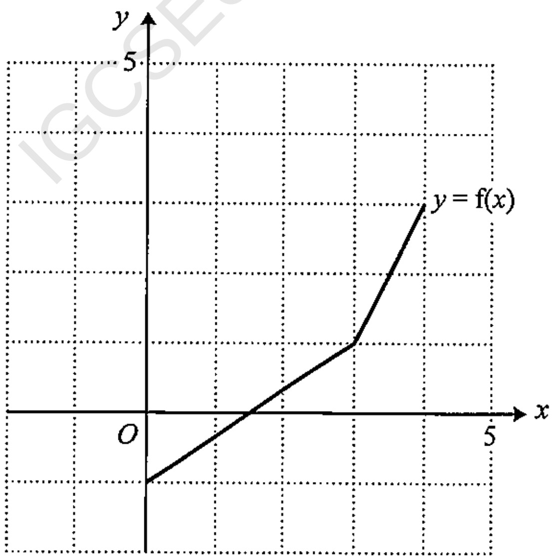
12 (a)



(i) Write down the letter of each graph which does **not** represent a function. [2]

(ii) Write down the letter of each graph which represents a function that does **not** have an inverse. [2]

(b)



The diagram shows the graph of a function $y = f(x)$. On the same axes sketch the graph of $y = f^{-1}(x)$. [2]

May Jun 2018 Code 22

13 **(a)** The graph of $y = k(3^x) + c$ passes through the points (0, 14) and (−2, 6). Find the value of k and of c. [3]

(b) The variables x and y are connected by the equation $y = e^x + 25 - 24e^{-x}$.

(i) Find the value of y when $x = 4$. [1]

(ii) Find the value of e^x when $y = 20$ and hence find the corresponding value of x. [4]

May Jun 2013 Code 22

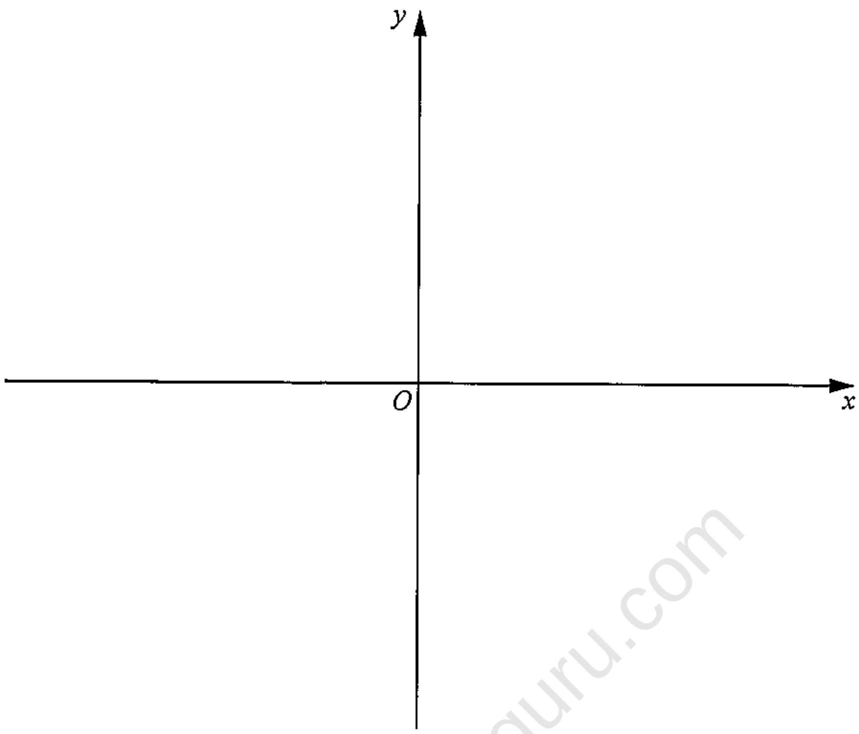
14 Find the set of values of x for which $x^2 < 6 - 5x$. [3]

Oct Nov 2013 Code 21,22

15 Find the set of values of x for which $x(x+2) < x$. [3]

May Jun 2014 Code 21

16 **(i)** On the axes below, sketch the graph of $y = |(x-4)(x+2)|$ showing the coordinates of the points where the curve meets the x-axis. [2]



(ii) Find the set of values of k for which $|(x-4)(x+2)| = k$ has four solutions. [3]

May Jun 2014 Code 21

17 **(i)** Express $2x^2 - x + 6$ in the form $p(x-q)^2 + r$, where p, q and r are constants to be found. [3]

(ii) Hence state the least value of $2x^2 - x + 6$ and the value of x at which this occurs. [2]

May Jun 2014 Code 21

18 The functions f and g are defined by

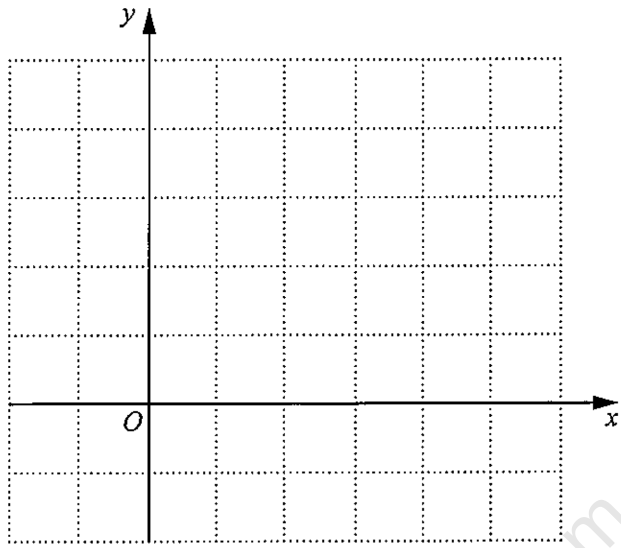
$$f(x) = \frac{2x}{x+1} \text{ for } x > 0,$$
$$g(x) = \sqrt{x+1} \text{ for } x > -1.$$

(i) Find fg(8). [2]

(ii) Find an expression for $f^2(x)$, giving your answer in the form $\frac{ax}{bx+c}$, where a, b and c are integers to be found. [3]

(iii) Find an expression for $g^{-1}(x)$, stating its domain and range. [4]

(iv) On the same axes, sketch the graphs of $y = g(x)$ and $y = g^{-1}(x)$, indicating the geometrical relationship between the graphs. [3]



May Jun 2014 Code 21

19 **(i)** Express $12x^2 - 6x + 5$ in the form $p(x-q)^2 + r$, where p, q and r are constants to be found. [3]

(ii) Hence find the greatest value of $\dfrac{1}{12x^2 - 6x + 5}$ and state the value of x at which this occurs. [2]

May Jun 2014 Code 22

20 The functions f and g are defined, for real values of x greater than 2, by

$$f(x) = 2^x - 1,$$
$$g(x) = x(x+1).$$

(i) State the range of f. [1]

(ii) Find an expression for $f^{-1}(x)$, stating its domain and range. [4]

(iii) Find an expression for $gf(x)$ and explain why the equation $gf(x) = 0$ has no solutions. [4]

May Jun 2014 Code 22

21 The function f is such that $f(x) = 2 + \sqrt{x-3}$ for $4 \leqslant x \leqslant 28$.

(i) Find the range of f. [2]

(ii) Find $f^2(12)$. [2]

(iii) Find an expression for $f^{-1}(x)$. [2]

The function g is defined by $g(x) = \dfrac{120}{x}$ for $x \geqslant 0$.

(iv) Find the value of x for which $gf(x) = 20$. [3]

May Jun 2014 Code 23

22 Solve the inequality $9x^2 + 2x - 1 < (x + 1)^2$. [3]

Oct Nov 2014 Code 21,22

23 The functions f and g are defined for real values of x by

$$f(x) = \sqrt{x-1} - 3 \qquad \text{for } x > 1,$$

$$g(x) = \frac{x-2}{2x-3} \qquad \text{for } x > 2.$$

(i) Find gf(37). [2]

(ii) Find an expression for $f^{-1}(x)$. [2]

(iii) Find an expression for $g^{-1}(x)$. [2]

Oct Nov 2014 Code 21,22

24 The number of bacteria B in a culture, t days after the first observation, is given by

$$B = 500 + 400e^{0.2t}.$$

(i) Find the initial number present. [1]

(ii) Find the number present after 10 days. [1]

(iii) Find the rate at which the bacteria are increasing after 10 days. [2]

(iv) Find the value of t when $B = 10000$. [3]

Oct Nov 2014 Code 21,22

25 The profit $\$P$ made by a company in its nth year is modelled by

$$P = 1000e^{an+b}.$$

In the first year the company made \$2000 profit.

(i) Show that $a + b = \ln 2$. [1]

In the second year the company made \$3297 profit.

(ii) Find another linear equation connecting a and b. [2]

(iii) Solve the two equations from parts **(i)** and **(ii)** to find the value of a and of b. [2]

(iv) Using your values for a and b, find the profit in the 10th year. [2]

Oct Nov 2014 Code 23

26 The functions f and g are defined for real values of x by

$$f(x) = \frac{2}{x} + 1 \text{ for } x > 1,$$

$$g(x) = x^2 + 2.$$

Find an expression for

(i) $f^{-1}(x)$, [2]

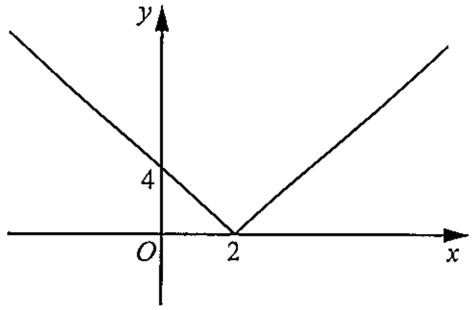
(ii) $gf(x)$, [2]

(iii) $fg(x)$. [2]

(iv) Show that $ff(x) = \frac{3x+2}{x+2}$ and solve $ff(x) = x$. [4]

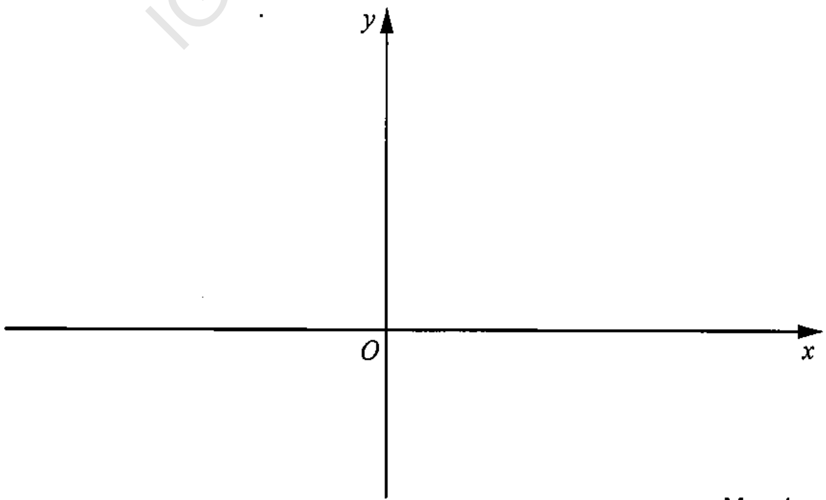
Oct Nov 2014 Code 33

27 (a)



The diagram shows the graph of $y = |f(x)|$ passing through $(0, 4)$ and touching the x-axis at $(2, 0)$. Given that the graph of $y = f(x)$ is a straight line, write down the two possible expressions for $f(x)$. [2]

(b) On the axes below, sketch the graph of $y = e^{-x} + 3$, stating the coordinates of any point of intersection with the coordinate axes. [3]



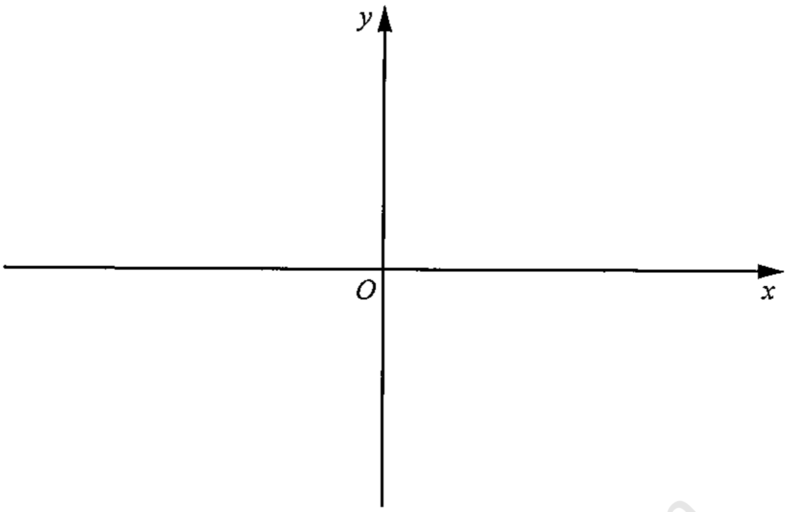
May Jun 2015 Code 21

28 (a) Find the set of values of x for which $4x^2 + 19x - 5 \leqslant 0$. [3]

(b) (i) Express $x^2 + 8x - 9$ in the form $(x + a)^2 + b$, where a and b are integers. [2]

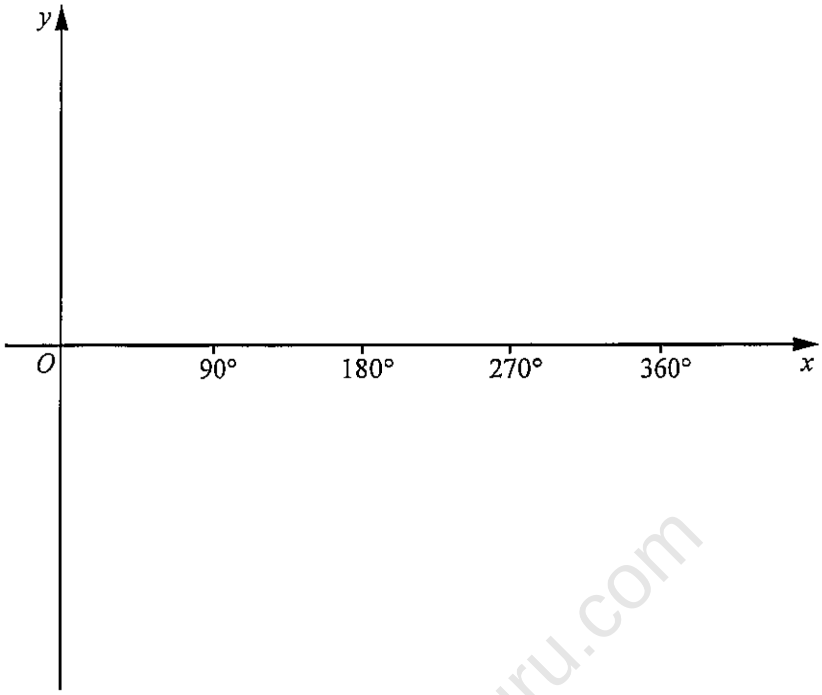
(ii) Use your answer to part **(i)** to find the greatest value of $9 - 8x - x^2$ and the value of x at which this occurs. [2]

(iii) Sketch the graph of $y = 9 - 8x - x^2$, indicating the coordinates of any points of intersection with the coordinate axes. [2]



May Jun 2015 Code 21,23

29 **(a)** The function f is defined by $f: x \mapsto |\sin x|$ for $0° \leqslant x \leqslant 360°$. On the axes below, sketch the graph of $y = f(x)$. [2]

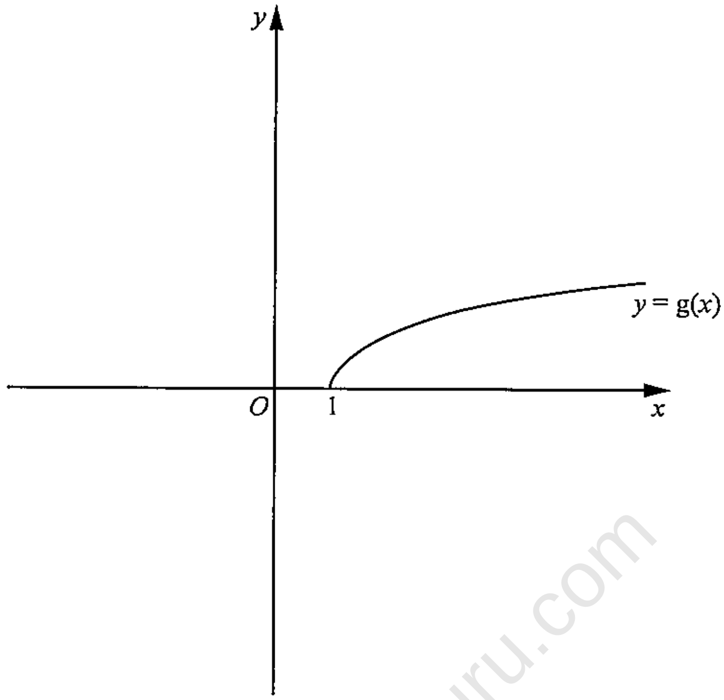


(b) The functions g and hg are defined, for $x \geqslant 1$, by

$$g(x) = \ln(4x - 3),$$
$$hg(x) = x.$$

(i) Show that $h(x) = \dfrac{e^x + 3}{4}$. [2]

(ii)



The diagram shows the graph of $y = g(x)$. Given that g and h are inverse functions, sketch, on the same diagram, the graph of $y = h(x)$. Give the coordinates of any point where your graph meets the coordinate axes. [2]

(iii) State the domain of h. [1]

(iv) State the range of h. [1]

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30 Given that $f(x) = 3x^2 + 12x + 2$,

(i) find values of a, b and c such that $f(x) = a(x + b)^2 + c$, [3]

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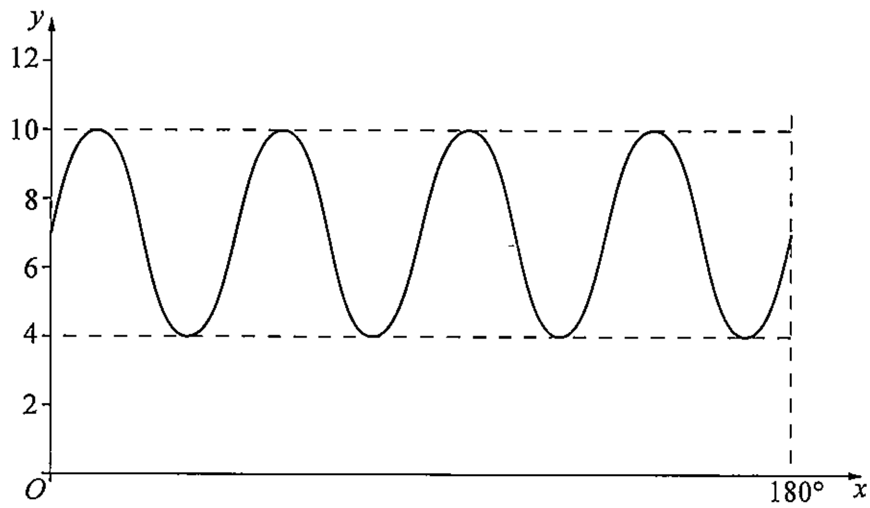
(ii) state the minimum value of $f(x)$ and the value of x at which it occurs, [2]

(iii) solve $f\left(\frac{1}{y}\right) = 0$, giving each answer for y correct to 2 decimal places. [3]

1 (a)



The diagram shows a sketch of the curve $y = a\sin(bx) + c$ for $0° \leqslant x \leqslant 180°$. Find the values of a, b and c. [3]

(b) Given that $f(x) = 5\cos 3x + 1$, for all x, state

(i) the period of f, [1]

(ii) the amplitude of f. [1]

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2 **(a)** Solve the equation

(i) $3\sin x - 5\cos x = 0$ for $0° < x < 360°$, [3]

(ii) $5\sin^2 y + 9\cos y - 3 = 0$ for $0° < y < 360°$. [5]

(b) Solve $\sin(3 - z) = 0.8$ for $0 < z < \pi$ radians. [4]

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3 **(i)** On the grid below draw, for $0° \leqslant x \leqslant 360°$, the graphs of $y = 3\sin 2x$ and $y = 2 + \cos x$. [4]

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(ii) State the number of values of x for which $3\sin 2x = 2 + \cos x$ in the interval $0° \leqslant x \leqslant 360°$. [1]

4 **(a)** Solve $4\sin x + 9\cos x = 0$ for $0° < x < 360°$. [3]

(b) Solve $\operatorname{cosec} y - 1 = 12\sin y$ for $0° < y < 360°$. [5]

(c) Solve $3\sec\left(\frac{z}{3}\right) = 5$ for $0 < z < 6\pi$ radians. [4]

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5 **(a)** Given that $\cos x = p$, find an expression, in terms of p, for $\tan^2 x$. [3]

(b) Prove that $(\cot\theta + \tan\theta)^2 = \sec^2\theta + \operatorname{cosec}^2\theta$. [3]

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6 Prove that $\left(\frac{1+\sin\theta}{\cos\theta}\right)^2 + \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = 2 + 4\tan^2\theta$. [4]

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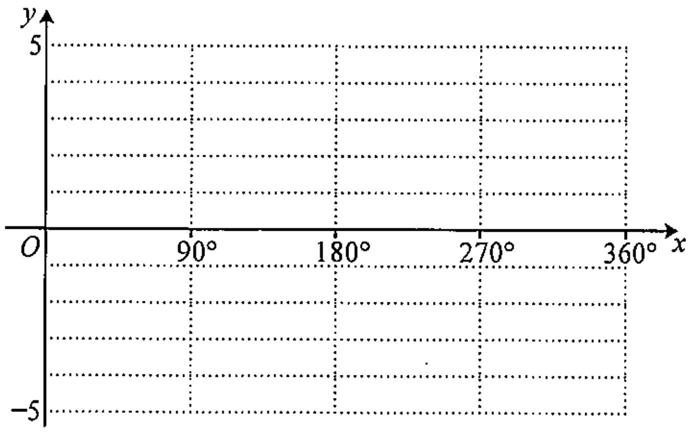
7 **(a)** The function f is defined, for $0° \leqslant x \leqslant 360°$, by $f(x) = 1 + 3\cos 2x$.

(i) Sketch the graph of $y = f(x)$ on the axes below. [4]



(ii) State the amplitude of f. [1]

(iii) State the period of f. [1]

(b) Given that $\cos x = p$, where $270° < x < 360°$, find $\operatorname{cosec} x$ in terms of p. [3]

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8 **(a)** Solve the equation $2\operatorname{cosec} x + \frac{7}{\cos x} = 0$ for $0° \leqslant x \leqslant 360°$. [4]

(b) Solve the equation $7\sin(2y - 1) = 5$ for $0 \leqslant y \leqslant 5$ radians. [5]

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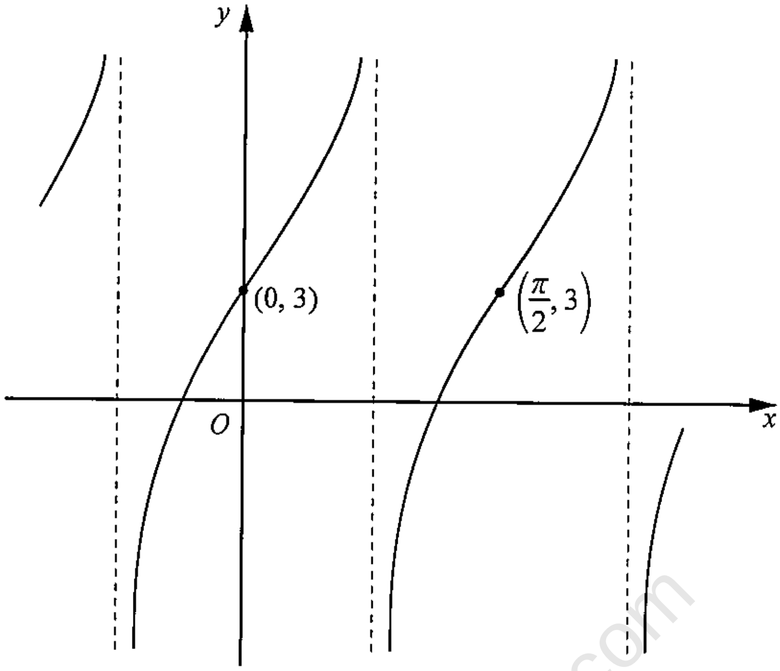
9



(a) (i) The diagram shows the graph of $y = A + C\tan(Bx)$ passing through the points (0, 3) and $\left(\frac{\pi}{2}, 3\right)$. Find the value of A and of B. [2]

(ii) Given that the point $\left(\frac{\pi}{8}, 7\right)$ also lies on the graph, find the value of C. [1]

(i) Prove that $\sec x \operatorname{cosec} x - \cot x = \tan x$. [4]

(ii) Use the result from part **(i)** to solve the equation $\sec x \operatorname{cosec} x = 3\cot x$ for $0° < x < 360°$. [4]

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10 **(i)** Prove that $\dfrac{1}{1-\cos x}+\dfrac{1}{1+\cos x}=2\operatorname{cosec}^2 x$. [3]

(ii) Hence solve the equation $\dfrac{1}{1-\cos x}+\dfrac{1}{1+\cos x}=8$ for $0° < x < 360°$. [4]

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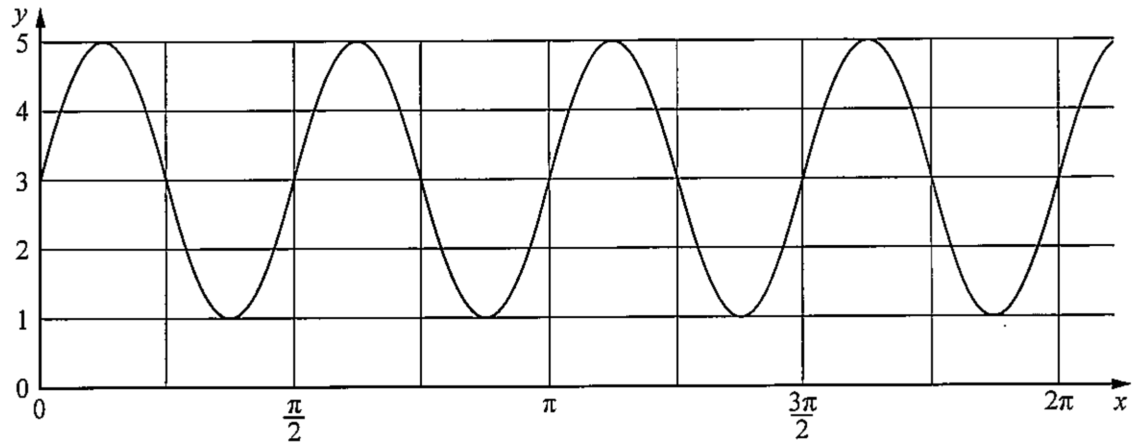
11



The figure shows part of the graph of $y = a + b\sin cx$.

(i) Find the value of each of the integers a, b and c. [3]

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Using your values of a, b and c find

(ii) $\frac{dy}{dx}$, [2]

(iii) the equation of the normal to the curve at $\left(\frac{\pi}{2}, 3\right)$. [3]

12 Solve the following equations.

(i) $4\sin 2x + 5\cos 2x = 0$ for $0° \leqslant x \leqslant 180°$ [3]

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(ii) $\cot^2 y + 3\operatorname{cosec} y = 3$ for $0° \leqslant y \leqslant 360°$ [5]

(iii) $\cos\left(z + \frac{\pi}{4}\right) = -\frac{1}{2}$ for $0 \leqslant z \leqslant 2\pi$ radians, giving each answer as a multiple of π [4]

13 **(i)** Prove that $\sec^2 x + \operatorname{cosec}^2 x = \sec^2 x \operatorname{cosec}^2 x$. [4]

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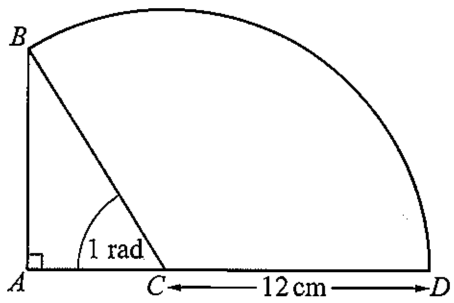
(ii) Hence, or otherwise, solve $\sec^2 x + \operatorname{cosec}^2 x = 4\tan^2 x$ for $90° < x < 270°$. [4]

1



The diagram shows a right-angled triangle ABC and a sector $CBDC$ of a circle with centre C and radius 12 cm. Angle $ACB = 1$ radian and ACD is a straight line.

(i) Show that the length of AB is approximately 10.1 cm. [1]

(ii) Find the perimeter of the shaded region. [5]

(iii) Find the area of the shaded region. [4]

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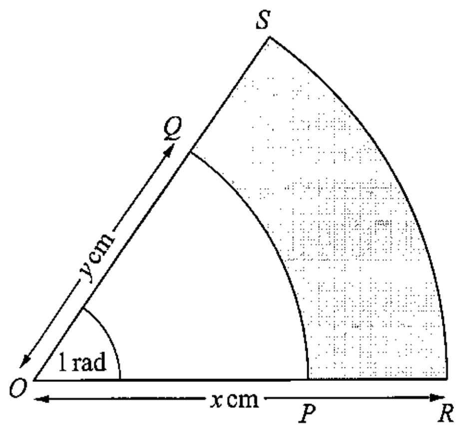
2



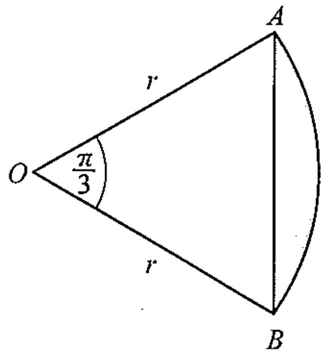
In the diagram PQ and RS are arcs of concentric circles with centre O and angle $POQ = 1$ radian. The radius of the larger circle is x cm and the radius of the smaller circle is y cm.

(i) Given that the perimeter of the shaded region is 20 cm, express y in terms of x. [2]

(ii) Given that the area of the shaded region is 16 cm^2, express y^2 in terms of x^2. [2]

(iii) Find the value of x and of y. [4]

3



The shaded region in the diagram is a segment of a circle with centre O and radius r cm.

Angle $AOB = \frac{\pi}{3}$ radians.

(i) Show that the perimeter of the segment is $r\left(\frac{3+\pi}{3}\right)$ cm. [2]

(ii) Given that the perimeter of the segment is 26 cm, find the value of r and the area of the segment. [5]

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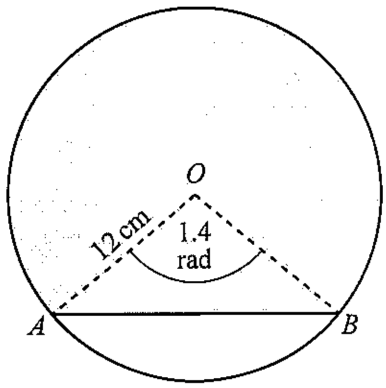
4



The diagram shows a circle with centre O and a chord AB. The radius of the circle is 12 cm and angle AOB is 1.4 radians.

(i) Find the perimeter of the shaded region. [5]

(ii) Find the area of the shaded region. [4]

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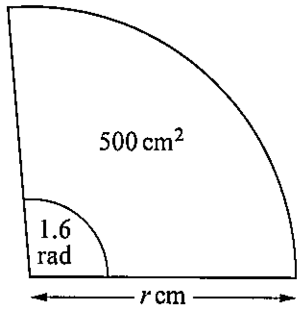
5



The diagram shows a sector of a circle of radius r cm. The angle of the sector is 1.6 radians and the area of the sector is $500\,\text{cm}^2$.

(i) Find the value of r. [2]

(ii) Hence find the perimeter of the sector. [2]

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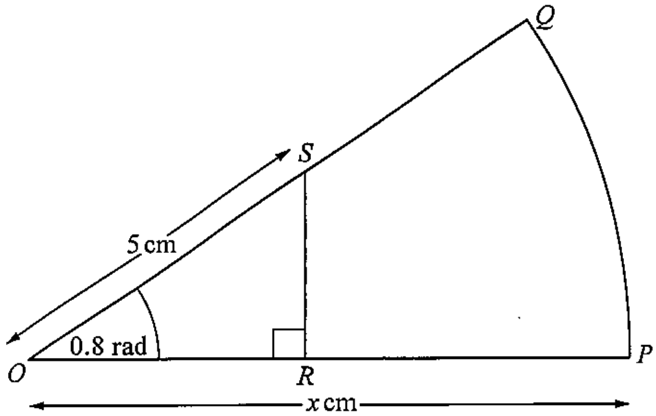
6



The diagram shows a sector OPQ of a circle with centre O and radius x cm. Angle POQ is 0.8 radians. The point S lies on OQ such that $OS = 5$ cm. The point R lies on OP such that angle ORS is a right angle. Given that the area of triangle ORS is one-fifth of the area of sector OPQ, find

(i) the area of sector OPQ in terms of x and hence show that the value of x is 8.837 correct to 4 significant figures, [5]

(ii) the perimeter of $PQSR$, [3]

(iii) the area of $PQSR$. [2]

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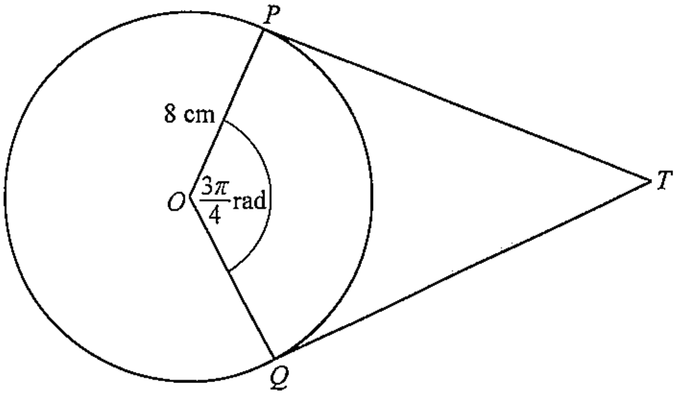
7



The diagram shows a circle, centre O, radius 8 cm. The points P and Q lie on the circle. The lines PT and QT are tangents to the circle and angle $POQ = \frac{3\pi}{4}$ radians.

(i) Find the length of PT. [2]

(ii) Find the area of the shaded region. [3]

(iii) Find the perimeter of the shaded region. [2]

May Jun 2015 Code 21,23

1 (a) A team of 7 people is to be chosen from 5 women and 7 men. Calculate the number of different ways in which this can be done if

(i) there are no restrictions, [1]

(ii) the team is to contain more women than men. [3]

(b) (i) How many different 4-digit numbers, less than 5000, can be formed using 4 of the 6 digits 1, 2, 3, 4, 5 and 6 if no digit can be used more than once? [2]

(ii) How many of these 4-digit numbers are divisible by 5? [2]

May Jun 2012 Code 22

2 **(a)** An art gallery displays 10 paintings in a row. Of these paintings, 5 are by Picasso, 4 by Monet and 1 by Turner.

(i) Find the number of different ways the paintings can be displayed if there are no restrictions. [1]

(ii) Find the number of different ways the paintings can be displayed if the paintings by each of the artists are kept together. [3]

(b) A committee of 4 senior students and 2 junior students is to be selected from a group of 6 senior students and 5 junior students.

(i) Calculate the number of different committees which can be selected. [3]

One of the 6 senior students is a cousin of one of the 5 junior students.

(ii) Calculate the number of different committees which can be selected if at most one of these cousins is included. [3]

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3 A 4-digit number is formed by using four of the six digits 2, 3, 4, 5, 6 and 8; no digit may be used more than once in any number. How many different 4-digit numbers can be formed if

(i) there are no restrictions, [2]

(ii) the number is even and more than 6000? [3]

Oct Nov 2012 Code 22

4 (i) Find how many different numbers can be formed using 4 of the digits 1, 2, 3, 4, 5, 6 and 7 if no digit is repeated. [1]

Find how many of these 4-digit numbers are

(ii) odd, [1]

(iii) odd and less than 3000. [3]

Oct Nov 2013 Code 23

5 A committee of four is to be selected from 7 men and 5 women. Find the number of different committees that could be selected if

(i) there are no restrictions, [1]

(ii) there must be two male and two female members. [2]

A brother and sister, Ken and Betty, are among the 7 men and 5 women.

(iii) Find how many different committees of four could be selected so that there are two male and two female members which must include either Ken or Betty but not both. [4]

Oct Nov 2014 Code 23

6 **(a)** A lock can be opened using only the number 4351. State whether this is a permutation or a combination of digits, giving a reason for your answer. [1]

(b) There are twenty numbered balls in a bag. Two of the balls are numbered 0, six are numbered 1, five are numbered 2 and seven are numbered 3, as shown in the table below.

Number on ball	0	1	2	3
Frequency	2	6	5	7

Four of these balls are chosen at random, without replacement. Calculate the number of ways this can be done so that

(i) the four balls all have the same number, [2]

(ii) the four balls all have different numbers, [2]

(iii) the four balls have numbers that total 3. [3]

May Jun 2015 Code 21,23

1 **(a)** Find the coefficient of x^3 in the expansion of

(i) $(1-2x)^7$, [2]

(ii) $(3+4x)(1-2x)^7$. [3]

(b) Find the term independent of x in the expansion of $\left(x+\frac{3}{x^2}\right)^6$. [3]

May Jun 2012 Code 22

2 **(i)** Find the coefficient of x^5 in the expansion of $(2-x)^8$. [2]

(ii) Find the coefficient of x^5 in the expansion of $(1+2x)(2-x)^8$. [3]

Oct Nov 2012 Code 22

3 (i) Find the first four terms in the expansion of $(2 + x)^6$ in ascending powers of x. [3]

(ii) Hence find the coefficient of x^3 in the expansion of $(1 + 3x)(1 - x)(2 + x)^6$. [4]

May Jun 2013 Code 21,23

4 **(a)** **(i)** Find the coefficient of x^3 in the expansion of $(1-2x)^6$. [2]

(ii) Find the coefficient of x^3 in the expansion of $\left(1+\frac{x}{2}\right)(1-2x)^6$. [3]

(b) Expand $\left(2\sqrt{x}+\frac{1}{\sqrt{x}}\right)^4$ in a series of powers of x with integer coefficients. [3]

Oct Nov 2013 Code 21,22

5 **(a)** Find the coefficient of x^5 in the expansion of $(3-2x)^8$. [2]

(b) **(i)** Write down the first three terms in the expansion of $(1+2x)^6$ in ascending powers of x. [2]

(ii) In the expansion of $(1+ax)(1+2x)^6$, the coefficient of x^2 is 1.5 times the coefficient of x. Find the value of the constant a. [4]

May Jun 2014 Code 21

6 **(i)** Find and simplify the first three terms of the expansion, in ascending powers of x, of $(1-4x)^5$. [2]

(ii) The first three terms in the expansion of $(1-4x)^5(1+ax+bx^2)$ are $1-23x+222x^2$. Find the value of each of the constants a and b. [4]

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7 In the expansion of $(1+2x)^n$, the coefficient of x^4 is ten times the coefficient of x^2. Find the value of the positive integer, n. [6]

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8 (i) Find, in the simplest form, the first 3 terms of the expansion of $(2-3x)^6$, in ascending powers of x. [3]

(ii) Find the coefficient of x^2 in the expansion of $(1+2x)(2-3x)^6$. [2]

Paper 2 - Oct Nov 2015 Code 21,22

1 Given that $f(x) = x^2 - \frac{648}{\sqrt{x}}$, find the value of x for which $f''(x) = 0$. [6]

May Jun 2012 Code 21,23

2 (i) Given that $y = \sqrt{(4x+1)^3}$, find $\frac{dy}{dx}$. [2]

(ii) Hence find the approximate increase in y as x increases from 6 to $6 + p$, where p is small. [2]

May Jun 2012 Code 22

3 An open rectangular cardboard box with a square base is to have a volume of 256 cm^3. Find the dimensions of the box if the area of cardboard used is as small as possible. [7]

May Jun 2012 Code 22

4 The normal to the curve $y = x^3 + 6x^2 - 34x + 44$ at the point $P\ (2, 8)$ cuts the x-axis at A and the y-axis at B. Show that the mid-point of the line AB lies on the line $4y = x + 9$. [8]

Oct Nov 2012 Code 21

5 The total surface area, A cm , of a solid cylinder with radius r cm and height 5 cm is given by $A = 2\pi r^2 + 10\pi r$. Given that r is increasing at a rate of $\frac{0.2}{\pi}$ cm s^{-1}, find the rate of increase of A when r is 6. [4]

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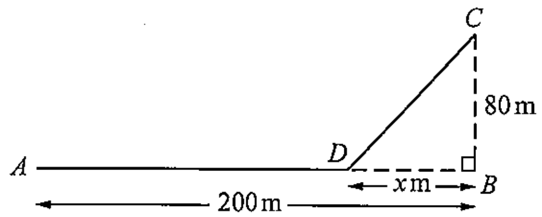
6



A track runs due east from A to B, a distance of 200 m. The point C is 80 m due north of B. A cyclist travels on the track from A to D, where D is x m due west of B. The cyclist then travels in a straight line across rough ground from D to C. The cyclist travels at $10\,\text{m s}^{-1}$ on the track and at $6\,\text{m s}^{-1}$ across rough ground.

(i) Show that the time taken, T s, for the cyclist to travel from A to C is given by

$$T = \frac{200 - x}{10} + \frac{\sqrt{(x^2 + 6400)}}{6}$$ [2]

(ii) Given that x can vary, find the value of x for which T has a stationary value and the corresponding value of T. [6]

Oct Nov 2012 Code 23

7 Variables x and y are related by the equation $y = 10 - 4\sin^2 x$, where $0 \leqslant x \leqslant \frac{\pi}{2}$. Given that x is increasing at a rate of 0.2 radians per second, find the corresponding rate of change of y when $y = 8$. [6]

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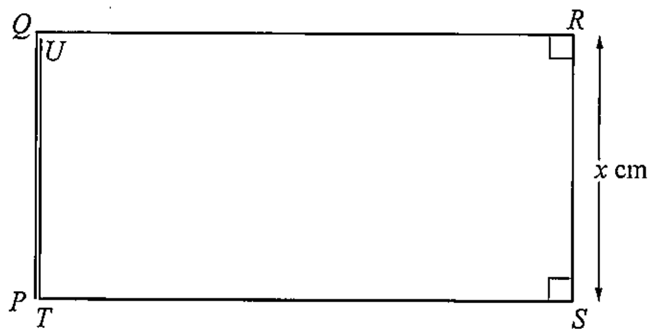
8



A piece of wire of length 96 cm is formed into the rectangular shape *PQRSTU* shown in the diagram. It is given that $PQ = TU = SR = x$ cm. It may be assumed that *PQ* and *TU* coincide and that *TS* and *QR* have the same length.

(i) Show that the area, A cm^2, enclosed by the wire is given by $A = \frac{96x - 3x^2}{2}$. [2]

(ii) Given that x can vary, find the stationary value of A and determine the nature of this stationary value. [4]

May Jun 2013 Code 21,23

9 Find the equation of the normal to the curve $y = \dfrac{x^2 + 8}{x - 2}$ at the point on the curve where $x = 4$. [6]

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10 Differentiate, with respect to x,

(i) $(3-5x)^{12}$, [2]

(ii) $x^2 \sin x$, [2]

(iii) $\dfrac{\tan x}{1+e^{2x}}$. [4]

May Jun 2013 Code 22

11 A curve has equation $y = 3x + \dfrac{1}{(x-4)^3}$.

(i) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [4]

(ii) Show that the coordinates of the stationary points of the curve are (5, 16) and (3, 8). [2]

(iii) Determine the nature of each of these stationary points. [2]

May Jun 2013 Code 22

12 **(i)** Given that $y = \left(\frac{1}{4}x - 5\right)^8$, find $\frac{dy}{dx}$. [2]

(ii) Hence find the approximate change in y as x increases from 12 to $12 + p$, where p is small. [2]

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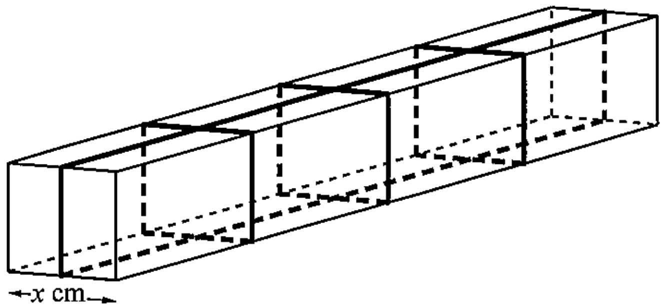
13



The diagram shows a box in the shape of a cuboid with a square cross-section of side x cm. The volume of the box is $3500\,cm^3$. Four pieces of tape are fastened round the box as shown. The pieces of tape are parallel to the edges of the box.

(i) Given that the total length of the four pieces of tape is L cm, show that $L = 14x + \frac{7000}{x^2}$. [3]

(ii) Given that x can vary, find the stationary value of L and determine the nature of this stationary value. [5]

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14 Find the coordinates of the stationary points on the curve $y = x^3 - 6x^2 - 36x + 16$. [5]

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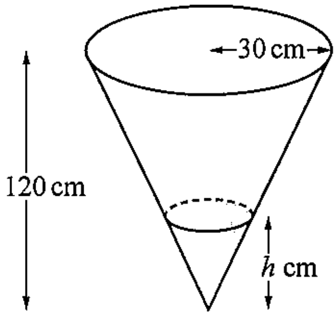
15



The volume of a cone of height H and radius R is

$$\frac{1}{3}\pi R^2 H$$

The diagram shows a container in the shape of a cone of height 120 cm and radius 30 cm. Water is poured into the container at a rate of $20\pi\,\text{cm}^3\,\text{s}^{-1}$.

(i) At the instant when the depth of water in the cone is h cm the volume of water in the cone is V cm^3. Show that $V = \frac{\pi h^3}{48}$. [3]

(ii) Find the rate at which h is increasing when $h = 50$. [3]

Oct Nov 2013 Code 23

(iii) Find the rate at which the circular area of the water's surface is increasing when $h = 50$. [4]

16 Given that a curve has equation $y = \frac{1}{x} + 2\sqrt{x}$, where $x > 0$, find

(i) $\frac{dy}{dx}$, [2]

(ii) $\frac{d^2y}{dx^2}$. [2]

Hence, or otherwise, find

(iii) the coordinates and nature of the stationary point of the curve. [4]

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17 A sector of a circle of radius r cm has an angle of θ radians, where $\theta < \pi$. The perimeter of the sector is 30 cm.

(i) Show that the area, A cm^2, of the sector is given by $A = 15r - r^2$. [3]

(ii) Given that r can vary, find the maximum area of the sector. [3]

May Jun 2014 Code 21

18 Find $\frac{dy}{dx}$ when

(i) $y = \cos 2x \sin\left(\frac{x}{3}\right)$, [4]

(ii) $y = \frac{\tan x}{1 + \ln x}$. [4]

May Jun 2014 Code 21

19 Differentiate with respect to x

(i) x^4e^{3x}, [2]

(ii) $\ln(2+\cos x)$, [2]

(iii) $\dfrac{\sin x}{1+\sqrt{x}}$. [3]

May Jun 2014 Code 22

20 A curve has equation $y = x^3 - 9x^2 + 24x$.

(i) Find the set of values of x for which $\frac{dy}{dx} \geqslant 0$. [4]

The normal to the curve at the point on the curve where $x = 3$ cuts the y-axis at the point P.

(ii) Find the equation of the normal and the coordinates of P. [5]

May Jun 2014 Code 22

21 **(i)** Determine the coordinates and nature of each of the two turning points on the curve $y = 4x + \frac{1}{x-2}$. [6]

(ii) Find the equation of the normal to the curve at the point (3, 13) and find the x-coordinate of the point where this normal cuts the curve again. [6]

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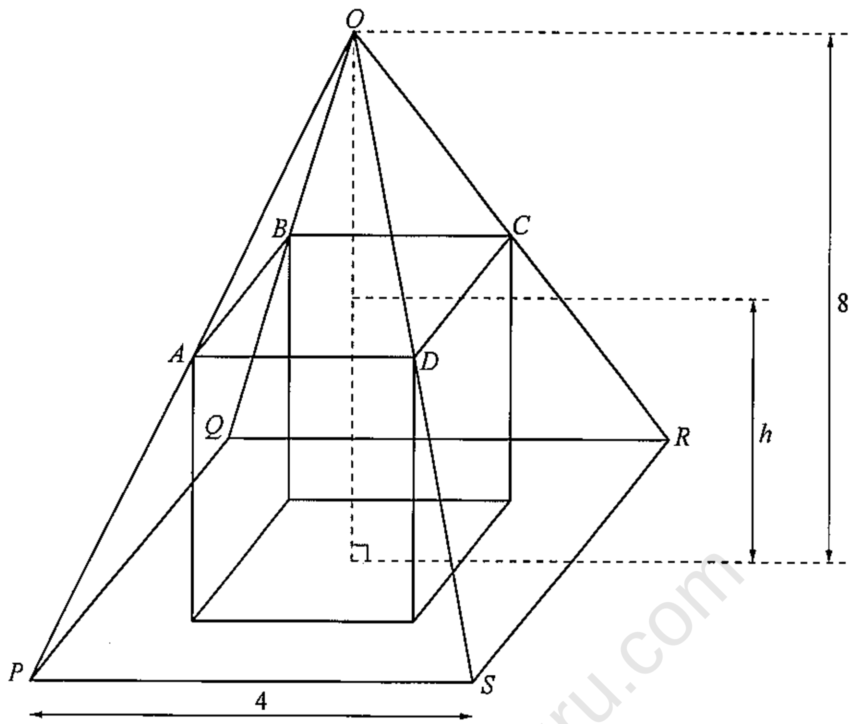
22



The diagram shows a cuboid of height h units inside a right pyramid $OPQRS$ of height 8 units and with square base of side 4 units. The base of the cuboid sits on the square base $PQRS$ of the pyramid. The points A, B, C and D are corners of the cuboid and lie on the edges OP, OQ, OR and OS, respectively, of the pyramid $OPQRS$. The pyramids $OPQRS$ and $OABCD$ are similar.

(i) Find an expression for AD in terms of h and hence show that the volume V of the cuboid is given by $V = \frac{h^3}{4} - 4h^2 + 16h$ units^3. [4]

(ii) Given that h can vary, find the value of h for which V is a maximum. [4]

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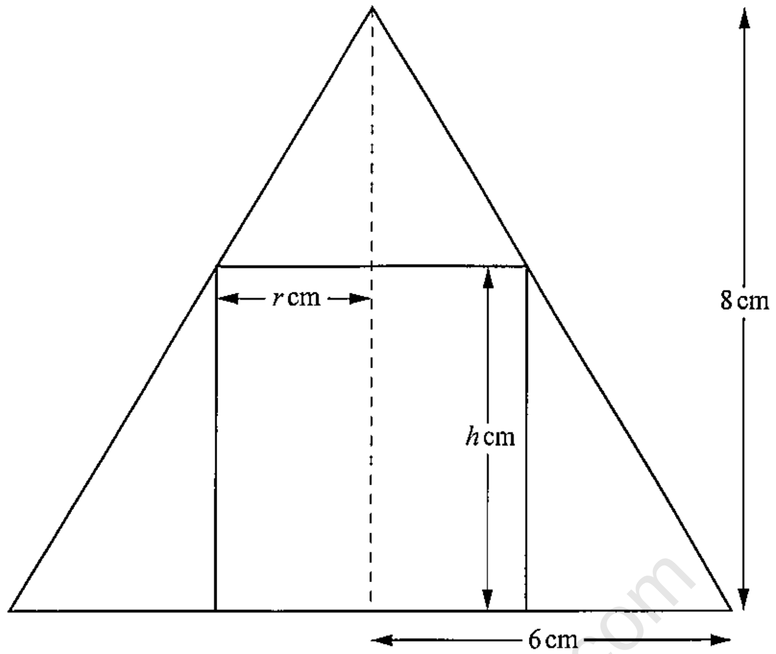

A cone, of height 8 cm and base radius 6 cm, is placed over a cylinder of radius r cm and height h cm and is in contact with the cylinder along the cylinder's upper rim. The arrangement is symmetrical and the diagram shows a vertical cross-section through the vertex of the cone.

(i) Use similar triangles to express h in terms of r. [2]

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(ii) Hence show that the volume, V cm^3, of the cylinder is given by $V = 8\pi r^2 - \frac{4}{3}\pi r^3$. [1]

(iii) Given that r can vary, find the value of r which gives a stationary value of V. Find this stationary value of V in terms of π and determine its nature. [6]

24 Find the equation of the tangent to the curve $y = x^3 + 3x^2 - 5x - 7$ at the point where $x = 2$. [5]

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25 **(a)** Given that $y = \frac{x^3}{2 - x^2}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

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(b) Given that $y = x\sqrt{4x + 6}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k(x + 1)}{\sqrt{4x + 6}}$ and state the value of k. [3]

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1 (i) Find $\frac{d}{dx}(x^2 \ln x)$. [2]

(ii) Hence, or otherwise, find $\int x \ln x \, dx$. [3]

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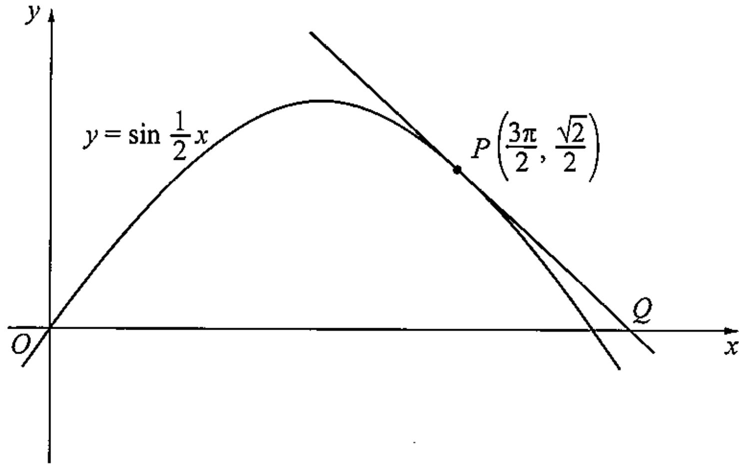
2



The diagram shows part of the curve $y = \sin \frac{1}{2}x$. The tangent to the curve at the point $P\left(\frac{3\pi}{2}, \frac{\sqrt{2}}{2}\right)$ cuts the x-axis at the point Q.

(i) Find the coordinates of Q. [4]

(ii) Find the area of the shaded region bounded by the curve, the tangent and the x-axis. [7]

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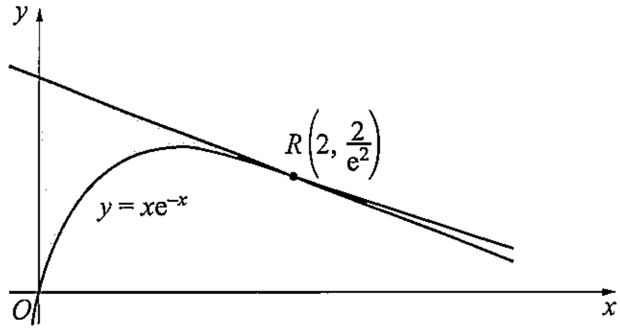
3 **(i)** Given that $y = xe^{-x}$, find $\frac{dy}{dx}$ and hence show that $\int xe^{-x}\,dx = -xe^{-x} - e^{-x} + c$. [4]



The diagram shows part of the curve $y = xe^{-x}$ and the tangent to the curve at the point $R\left(2, \frac{2}{e^2}\right)$.

(ii) Find the area of the shaded region bounded by the curve, the tangent and the y-axis. [7]

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4 (i) Find $\frac{d}{dx}(\tan 4x)$. [2]

(ii) Hence find $\int (1 + \sec^2 4x)\,dx$. [3]

(iii) Hence show that $\int_{-\frac{\pi}{16}}^{\frac{\pi}{16}} (1 + \sec^2 4x)\,dx = k(\pi+4)$, where k is a constant to be found. [2]

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5 The point $A(0, 10)$ lies on the curve for which $\frac{dy}{dx} = e^{-\frac{x}{4}}$. The point B, with x-coordinate -4, also lies on the curve.

(i) Find, in terms of e, the y-coordinate of B. [5]

The tangents to the curve at the points A and B intersect at the point C.

(ii) Find, in terms of e, the x-coordinate of the point C. [5]

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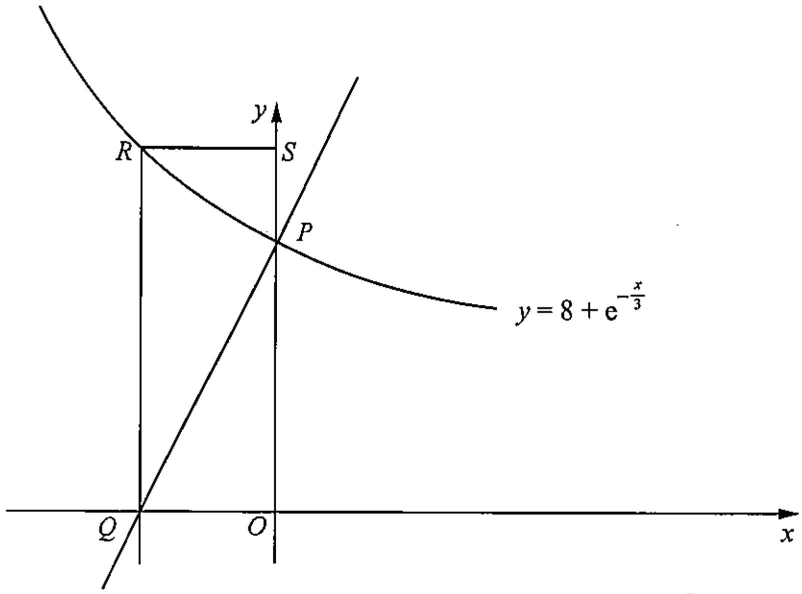
6



The diagram shows part of the curve $y = 8 + e^{-\frac{x}{3}}$ crossing the y-axis at P. The normal to the curve at P meets the x-axis at Q.

(i) Find the coordinates of Q. [4]

The line through Q, parallel to the y-axis, meets the curve at R and $OQRS$ is a rectangle.

(ii) Find $\int \left(8 + e^{-\frac{x}{3}}\right) dx$ and hence find the area of the shaded region. [6]

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7 **(a)** Find $\int (x + 3)\sqrt{x}\,dx$. [3]

(b) Find $\int \frac{20}{(2x + 5)^2}\,dx$ and hence evaluate $\int_0^{10} \frac{20}{(2x + 5)^2}\,dx$. [4]

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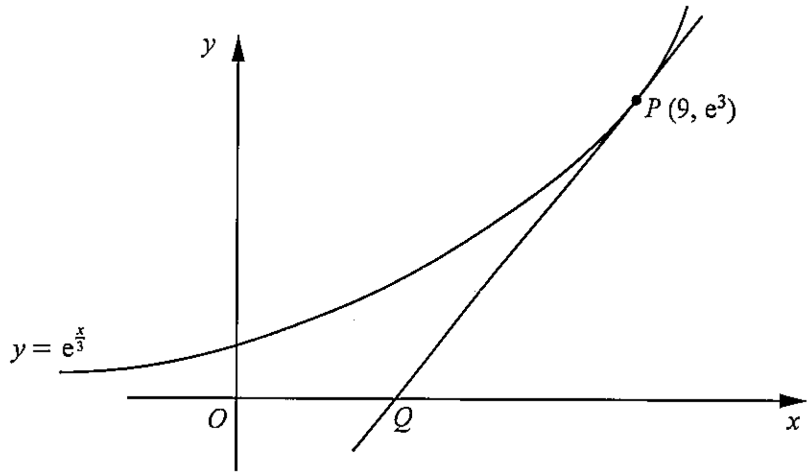

The diagram shows part of the curve $y = e^{\frac{x}{3}}$. The tangent to the curve at $P\,(9, e^3)$ meets the x-axis at Q.

(i) Find the coordinates of Q. [4]

(ii) Find the area of the shaded region bounded by the curve, the coordinate axes and the tangent to the curve at P. [6]

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9 A curve is such that $\frac{dy}{dx} = (2x+1)^{\frac{1}{2}}$. The curve passes through the point (4, 10).

(i) Find the equation of the curve. [4]

(ii) Find $\int y\,dx$ and hence evaluate $\int_0^{1.5} y\,dx$. [5]

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10 A curve is such that $\frac{dy}{dx} = 6x^2 - 8x + 3$.

(i) Show that the curve has no stationary points. [2]

Given that the curve passes through the point $P(2, 10)$,

(ii) find the equation of the tangent to the curve at the point P, [2]

(iii) find the equation of the curve. [4]

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11 **(i)** Given that $y = \frac{2x}{\sqrt{x^2+21}}$, show that $\frac{dy}{dx} = \frac{k}{\sqrt{(x^2+21)^3}}$, where k is a constant to be found. [5]

(ii) Hence find $\int \frac{6}{\sqrt{(x^2+21)^3}}dx$ and evaluate $\int_2^{10} \frac{6}{\sqrt{(x^2+21)^3}}dx$. [3]

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12 **(i)** Given that $y = \dfrac{x^2}{2 + x^2}$, show that $\dfrac{dy}{dx} = \dfrac{kx}{(2 + x^2)^2}$, where k is a constant to be found. [3]

(ii) Hence find $\displaystyle\int \frac{x}{(2 + x^2)^2}\,dx$. [2]

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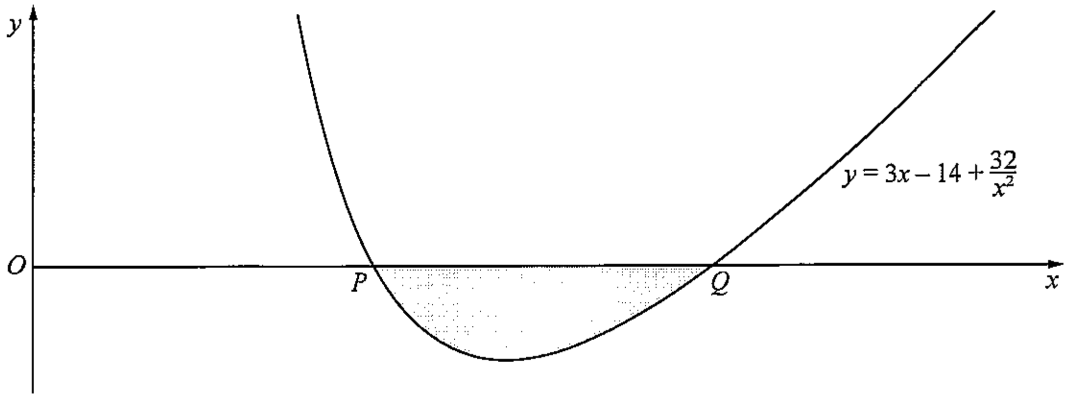
13 The diagram below shows part of the curve $y = 3x - 14 + \frac{32}{x^2}$ cutting the x-axis at the points P and Q.



(iii) State the x-coordinates of P and Q. [1]

(iv) Find $\int (3x - 14 + \frac{32}{x^2})\,dx$ and hence determine the area of the shaded region. [4]

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14 **(a)** (i) Find $\int e^{4x+3}\,dx$. [2]

(ii) Hence evaluate $\int_{2.5}^{3} e^{4x+3}\,dx$. [2]

(b) (i) Find $\int \cos\left(\frac{x}{3}\right)dx$. [2]

(ii) Hence evaluate $\int_{0}^{\frac{\pi}{6}} \cos\left(\frac{x}{3}\right)dx$. [2]

(c) Find $\int (x^{-1}+x)^2\,dx$. [4]

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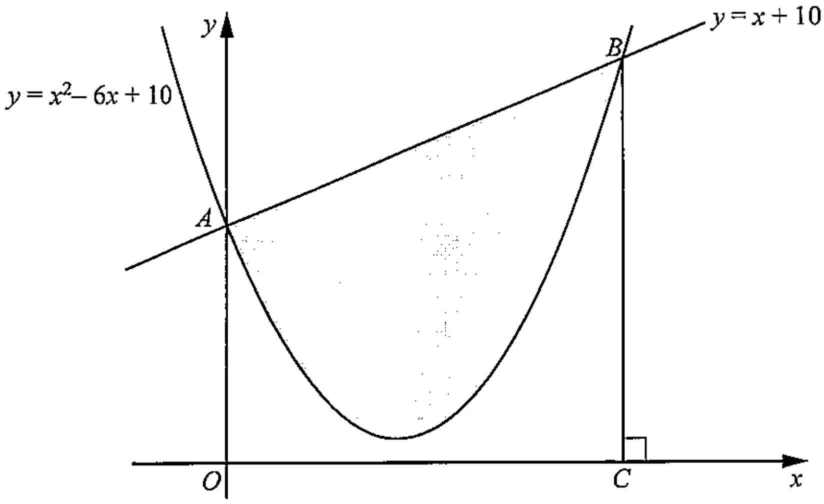
15



The graph of $y = x^2 - 6x + 10$ cuts the y-axis at A. The graphs of $y = x^2 - 6x + 10$ and $y = x + 10$ cut one another at A and B. The line BC is perpendicular to the x-axis. Calculate the area of the shaded region enclosed by the curve and the line AB, showing all your working. [8]

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16 **(i)** Given that $\frac{d}{dx}(e^{2-x^2}) = kxe^{2-x^2}$, state the value of k. [1]

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(ii) Using your result from part **(i)**, find $\int 3xe^{2-x^2}dx$. [2]

(iii) Hence find the area enclosed by the curve $y = 3xe^{2-x^2}$, the x-axis and the lines $x = 1$ and $x = \sqrt{2}$. [2]

(iv) Find the coordinates of the stationary points on the curve $y = 3xe^{2-x^2}$. [4]

1 A particle moves in a straight line so that, t s after passing through a fixed point O, its velocity, $v\,\mathrm{ms}^{-1}$, is given by $v = 2t - 11 + \dfrac{6}{t+1}$. Find the acceleration of the particle when it is at instantaneous rest. [7]

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2 A particle travels in a straight line so that, t s after passing through a fixed point O, its displacement, s m, from O is given by $s = t^2 - 10t + 10\ln(1 + t)$, where $t > 0$.

(i) Find the distance travelled in the twelfth second. [2]

(ii) Find the value of t when the particle is at instantaneous rest. [5]

(iii) Find the acceleration of the particle when $t = 9$. [3]

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3 A particle travels in a straight line so that, t s after passing through a fixed point O, its velocity, $v\,\text{cms}^{-1}$, is given by $v = 4e^{2t} - 24t$.

(i) Find the velocity of the particle as it passes through O. [1]

(ii) Find the distance travelled by the particle in the third second. [4]

(iii) Find an expression for the acceleration of the particle and hence find the stationary value of the velocity. [5]

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4 The acceleration, a m s^{-2}, of a particle, t s after passing through a fixed point O, is given by $a = 4 - 2t$, for $t > 0$. The particle, which moves in a straight line, passes through O with a velocity of 12 m s^{-1}.

(i) Find the value of t when the particle comes to instantaneous rest. [5]

(ii) Find the distance from O of the particle when it comes to instantaneous rest. [3]

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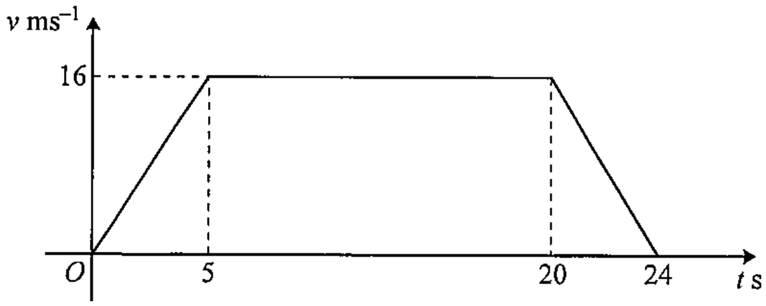

The velocity-time graph represents the motion of a particle moving in a straight line.

(i) Find the acceleration during the first 5 seconds. [1]

(ii) Find the length of time for which the particle is travelling with constant velocity. [1]

(iii) Find the total distance travelled by the particle. [3]

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6 A particle travels in a straight line so that, t s after passing through a fixed point O, its velocity, $v\,\text{ms}^{-1}$, is given by $v = 3 + 6\sin 2t$.

(i) Find the velocity of the particle when $t = \frac{\pi}{4}$. [1]

(ii) Find the acceleration of the particle when $t = 2$. [3]

The particle first comes to instantaneous rest at the point P.

(iii) Find the distance OP. [5]

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7 A particle moving in a straight line passes through a fixed point O. The displacement, x metres, of the particle, t seconds after it passes through O, is given by $x = t + 2\sin t$.

(i) Find an expression for the velocity, $v\,\text{ms}^{-1}$, at time t. [2]

When the particle is first at instantaneous rest, find

(ii) the value of t, [2]

(iii) its displacement and acceleration. [3]

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8 A particle moving in a straight line passes through a fixed point O. The displacement, x metres, of the particle, t seconds after it passes through O, is given by $x = 5t - 3\cos 2t + 3$.

(i) Find expressions for the velocity and acceleration of the particle after t seconds. [3]

(ii) Find the maximum velocity of the particle and the value of t at which this first occurs. [3]

(iii) Find the value of t when the velocity of the particle is first equal to $2\,\mathrm{ms}^{-1}$ and its acceleration at this time. [3]

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9 A particle P is projected from the origin O so that it moves in a straight line. At time t seconds after projection, the velocity of the particle, v ms^{-1}, is given by $v = 2t^2 - 14t + 12$.

(i) Find the time at which P first comes to instantaneous rest. [2]

(ii) Find an expression for the displacement of P from O at time t seconds. [3]

(iii) Find the acceleration of P when $t = 3$. [2]

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10 A particle is moving in a straight line such that its velocity, $v\,\text{ms}^{-1}$, t seconds after passing a fixed point O is $v = e^{2t} - 6e^{-2t} - 1$.

(i) Find an expression for the displacement, s m, from O of the particle after t seconds. [3]

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(ii) Using the substitution $u = e^{2t}$, or otherwise, find the time when the particle is at rest. [3]

(iii) Find the acceleration at this time. [2]

11 The velocity, v ms^{-1}, of a particle travelling in a straight line, t seconds after passing through a fixed point O, is given by $v = \dfrac{10}{(2+t)^2}$.

(i) Find the acceleration of the particle when $t = 3$. [3]

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(ii) Explain why the particle never comes to rest. [1]

(iii) Find an expression for the displacement of the particle from O after time t s. [3]

(iv) Find the distance travelled by the particle between $t = 3$ and $t = 8$. [2]

1 Relative to an origin O, the position vectors of the points A and B are $2\mathbf{i} - 3\mathbf{j}$ and $11\mathbf{i} + 42\mathbf{j}$ respectively.

(i) Write down an expression for $\overrightarrow{AB}$. [2]

The point C lies on AB such that $\overrightarrow{AC} = \frac{1}{3}\overrightarrow{AB}$.

(ii) Find the length of $\overrightarrow{OC}$. [4]

The point D lies on $\overrightarrow{OA}$ such that $\overrightarrow{DC}$ is parallel to $\overrightarrow{OB}$.

(iii) Find the position vector of D. [2]

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2 The points X, Y and Z are such that $\overrightarrow{XY} = 3\overrightarrow{YZ}$. The position vectors of X and Z, relative to an origin O, are $\begin{pmatrix} 4 \\ -27 \end{pmatrix}$ and $\begin{pmatrix} 20 \\ -7 \end{pmatrix}$ respectively. Find the unit vector in the direction $\overrightarrow{OY}$. [5]

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3 The position vectors of the points A and B, relative to an origin O, are $4\mathbf{i} - 21\mathbf{j}$ and $22\mathbf{i} - 30\mathbf{j}$ respectively. The point C lies on AB such that $\overrightarrow{AB} = 3\overrightarrow{AC}$.

(i) Find the position vector of C relative to O. [4]

(ii) Find the unit vector in the direction $\overrightarrow{OC}$. [2]

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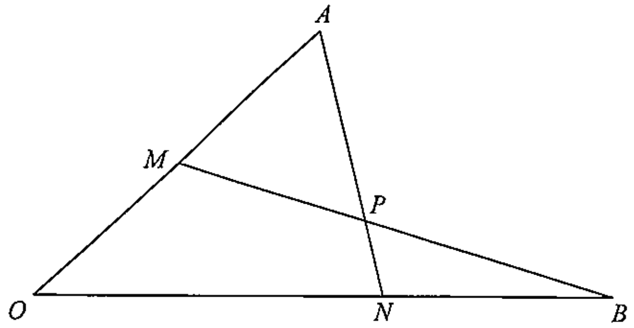
4



In the diagram $\overrightarrow{OA} = 2\mathbf{a}$ and $\overrightarrow{OB} = 5\mathbf{b}$. The point M is the midpoint of OA and the point N lies on OB such that $ON:NB = 3:2$.

(i) Find an expression for the vector $\overrightarrow{MB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

The point P lies on AN such that $\overrightarrow{AP} = \lambda\overrightarrow{AN}$.

(ii) Find an expression for the vector $\overrightarrow{AP}$ in terms of λ, $\mathbf{a}$ and $\mathbf{b}$. [2]

(iii) Find an expression for the vector $\overrightarrow{MP}$ in terms of λ, **a** and **b**. [2]

(iv) Given that M, P and B are collinear, find the value of λ. [4]

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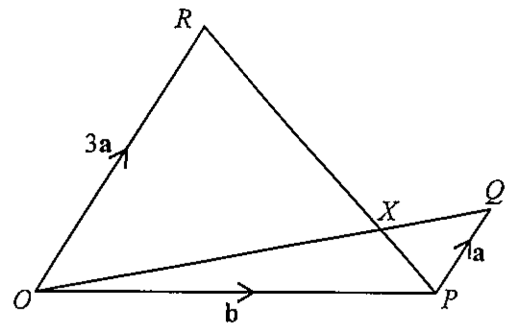

In the diagram $\overrightarrow{OP} = \mathbf{b}$, $\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{OR} = 3\mathbf{a}$. The lines OQ and PR intersect at X.

(i) Given that $\overrightarrow{OX} = \mu\overrightarrow{OQ}$, express $\overrightarrow{OX}$ in terms of μ, $\mathbf{a}$ and $\mathbf{b}$. [1]

(ii) Given that $\overrightarrow{RX} = \lambda\overrightarrow{RP}$, express $\overrightarrow{OX}$ in terms of λ, $\mathbf{a}$ and $\mathbf{b}$. [2]

(iii) Hence find the value of μ and of λ and state the value of the ratio $\frac{RX}{XP}$. [3]

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6 (a) The four points O, A, B and C are such that

$$\overrightarrow{OA} = 5\mathbf{a}, \quad \overrightarrow{OB} = 15\mathbf{b}, \quad \overrightarrow{OC} = 24\mathbf{b} - 3\mathbf{a}.$$

Show that B lies on the line AC. [3]

(b) Relative to an origin O, the position vector of the point P is $\mathbf{i} - 4\mathbf{j}$ and the position vector of the point Q is $3\mathbf{i} + 7\mathbf{j}$. Find

(i) $|\overrightarrow{PQ}|$, [2]

(ii) the unit vector in the direction $\overrightarrow{PQ}$, [1]

(iii) the position vector of M, the mid-point of PQ. [2]

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1 In this question $\begin{pmatrix}1\\0\end{pmatrix}$ is a unit vector due east and $\begin{pmatrix}0\\1\end{pmatrix}$ is a unit vector due north. At 12 00 a coastguard, at point O, observes a ship with position vector $\begin{pmatrix}16\\12\end{pmatrix}$ km relative to O. The ship is moving at a steady speed of 10kmh^{-1} on a bearing of 330°.

(i) Find the value of p such that $\begin{pmatrix}-5\\p\end{pmatrix}$ kmh^{-1} represents the velocity of the ship. [2]

(ii) Write down, in terms of t, the position vector of the ship, relative to O, t hours after 12 00. [2]

(iii) Find the time when the ship is due north of O. [2]

(iv) Find the distance of the ship from O at this time. [2]

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2 A plane, whose speed in still air is $420\,\text{km}\,\text{h}^{-1}$, travels directly from A to B, a distance of 1000 km. The bearing of B from A is 230° and there is a wind of $80\,\text{km}\,\text{h}^{-1}$ from the east.

(i) Find the bearing on which the plane was steered. [4]

(ii) Find the time taken for the journey. [4]

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3 A plane, whose speed in still air is $240\,\text{kmh}^{-1}$, flies directly from A to B, where B is 500 km from A on a bearing of 032°. There is a constant wind of $50\,\text{kmh}^{-1}$ blowing from the west.

(i) Find the bearing on which the plane is steered. [4]

(ii) Find, to the nearest minute, the time taken for the flight. [4]

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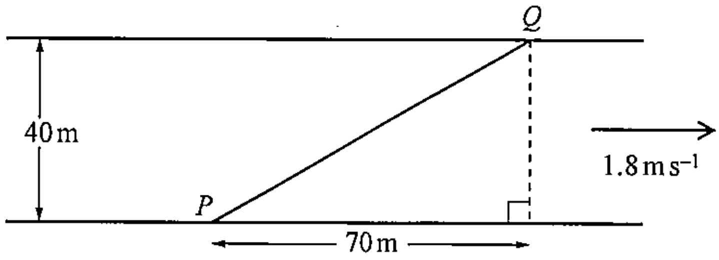
4



The diagram shows a river with parallel banks. The river is 40 m wide and is flowing with a speed of 1.8 m s^{-1}. A canoe travels in a straight line from a point P on one bank to a point Q on the opposite bank 70 m downstream from P. Given that the canoe takes 12 s to travel from P to Q, calculate the speed of the canoe in still water and the angle to the bank that the canoe was steered. [8]

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5 In this question $\mathbf{i}$ is a unit vector due east and $\mathbf{j}$ is a unit vector due north.

At time $t = 0$ boat A leaves the origin O and travels with velocity $(2\mathbf{i} + 4\mathbf{j})\,\text{km h}^{-1}$. Also at time $t = 0$ boat B leaves the point with position vector $(-21\mathbf{i} + 22\mathbf{j})\,\text{km}$ and travels with velocity $(5\mathbf{i} + 3\mathbf{j})\,\text{km h}^{-1}$.

(i) Write down the position vectors of boats A and B after t hours. [2]

(ii) Show that A and B are 25 km apart when $t = 2$. [3]

(iii) Find the length of time for which A and B are less than 25 km apart. [5]

6 A river, which is 80 m wide, flows at $2\,ms^{-1}$ between parallel, straight banks. A man wants to row his boat straight across the river and land on the other bank directly opposite his starting point. He is able to row his boat in still water at $3\,ms^{-1}$. Find

(i) the direction in which he must row his boat, [2]

(ii) the time it takes him to cross the river. [3]

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7 Relative to an origin O, points A, B and C have position vectors $\begin{pmatrix}5\\4\end{pmatrix}$, $\begin{pmatrix}-10\\12\end{pmatrix}$ and $\begin{pmatrix}6\\-18\end{pmatrix}$ respectively. All distances are measured in kilometres. A man drives at a constant speed directly from A to B in 20 minutes.

(i) Calculate the speed in kmh^{-1} at which the man drives from A to B. [3]

He now drives directly from B to C at the same speed.

(ii) Find how long it takes him to drive from B to C. [3]

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8 A plane that can travel at 250 kmh^{-1} in still air sets off on a bearing of 070°. A wind with speed w kmh^{-1} from the south blows the plane off course so that the plane actually travels on a bearing of 060°.

Find, in kmh^{-1}, the resultant speed V of the plane and the windspeed w. [5]

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